Learning Outcomes based Curriculum Framework (LOCF)

B.A./B.Sc. (Hons) Mathematics
&
B.A./B.Sc. with Mathematics as a Subject
2019

ज्ञान-विज्ञान विमुक्तये

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Table of Contents

- **Preamble** i

1. **Introduction** 1
2. **Learning Outcomes-based approach to Curriculum Planning** 2
   2.1. Nature and extent of Bachelor's Degree Programme 3
   2.2. Aims of Bachelor's degree programme in Mathematics 3
   2.3. Key outcomes underpinning curriculum planning and development 4
3. **Graduate Attributes in Mathematics** 5
   3.1. Disciplinary knowledge 5
   3.2. Communications skills 5
   3.3. Critical thinking and analytical reasoning 5
   3.4. Problem solving 5
   3.5. Research-related skills 6
   3.6. Information/digital literacy 6
   3.7. Self-directed learning 6
   3.8. Moral and ethical awareness/reasoning 6
   3.9. Lifelong learning 6
4. **Qualification descriptors for B.A./B.Sc. (Hons) Mathematics and B.A./B.Sc. with Mathematics as a subject** 6
5. **Programme Learning Outcomes of B.A./B.Sc. (Hons) Mathematics and** 8
   B.A./B.Sc. with Mathematics as a Subject 8
6. **Structure of B.A./B.Sc. Mathematics** 8
   6.1. Course learning outcomes 8
      6.1.1. Credit distribution for B.A./B.Sc. (Hons) Mathematics 11
      6.1.2. Credit distribution for B.A./B.Sc. with Mathematics as a subject 11
   6.2. Contents for each course 11
      6.2.1. Contents of courses for B.A./B.Sc. (Hons) Mathematics 11
      6.2.2. Contents of courses for B.A./B.Sc. with Mathematics as a subject 67
   6.3. References for each course 103
7. **Teaching-Learning Process** 103
8. **Assessment Methods** 105
9. **Keywords** 105

Preamble

The LOCF (Learning Outcomes based Curriculum Framework) committee constituted by University Grants Commission (UGC) is pleased to submit its report concerning the syllabi for B.A./B.Sc. (Honours) Mathematics and B.A./B.Sc. with Mathematics as a subject. The committee discussed the framework of syllabi in its meetings and suggests the implementation of these syllabi in the Departments/Schools of Mathematics in Universities/Colleges/Institutes based on following facts:

1. The learning outcomes of each paper are designed so that these may help learners to understand the main objectives of studying the course.

2. This will enable learners to select elective papers depending on the individual inclinations and contemporary requirements.

3. The objectives of LOCF are to mentally prepare the students to learn Mathematics leading to graduate degree with honours in Mathematics or with Mathematics as a subject.

4. These syllabi in Mathematics under CBCS are recommended keeping in view of the wide applications of Mathematics in science, engineering, social science, business and a host of other areas.

5. The study of the syllabi will enable the students to be equipped with the state of the art of the subject and will empower them to get jobs in technological and engineering fields as well as in business, education and healthcare sectors.

6. The LOCF committee in Mathematics has prepared this draft paying suitable attention to objectives and learning outcomes of the papers. These syllabi may be implemented with minor modifications with appropriate justifications keeping in view regional, national and international context and needs.

7. The outcomes of each paper may be modified as per the local requirements.

8. The text books mentioned in references are denotative/demonstrative. The divisions of each paper in units are specified to the context mentioned in courses. These units will help the learners to complete the study of concerned paper in certain periods and prepare them for examinations.

9. The papers are organized considering the credit load in a particular semester. The core papers of general interest are suggested for semesters I to IV. The elective courses and advanced courses are proposed for the B.A./B.Sc. (Hons.) students of semesters V & VI and the elective courses for the students of B.A./B.Sc. semesters V & VI having Mathematics as a subject.

10. The mathematics is a vast subject with immense diversity. Hence it is very difficult for every student to learn each branch of mathematics, even though each paper has its unique importance.

Under these circumstances, LOCF in Mathematics suggests a number of elective papers along with compulsory papers. A student can select elective papers as per her/his needs and interests.

11. The committee expects that the papers may be taught using various Computer Algebra Systems (CAS) softwares such as Mathematica, MATLAB, Maxima and R to strengthen the conceptual understanding and to widen up the horizon of students' self-experience.

12. The committee of the LOCF in Mathematics expects that the concerned departments/colleges/institutes/universities will encourage their faculty members to include necessary topics in addition to courses suggested by LOCF committee. It is hoped that the needs of all round development in the careers of learners/students will be fulfilled by the recommendations of LOCF in Mathematics.

Learning Outcomes-based Curriculum Framework in B.A./B.Sc. (Hons) Mathematics & B.A./B.Sc. with Mathematics as a subject

1. Introduction

One of the significant reforms in the undergraduate education is to introduce the Learning Outcomes-based Curriculum Framework (LOCF) which makes it student-centric, interactive and outcome-oriented with well-defined aims, objectives and goals to achieve. Outcome based learning is the principal end of pedagogical transactions in higher education in today's world in the light of exponential changes brought about in science and technology, especially in mathematics, and the prevalent utilitarian world view of the society. The learning outcomes are attained by students through skills acquired during a programme of study. Programme learning outcomes will include subject-specific skills and generic skills, including transferable global skills and competencies. It would also focus on knowledge and skills that prepare students for further study, employment, and citizenship. They help ensure comparability of learning levels and academic standards across colleges/universities and provide a broad picture of the level of competence of graduates.

The quality education in a subject like mathematics is a very challenging task for Higher Education System in India. UGC has already taken an appropriate measure to define the minimum levels of learning for mathematics courses for undergraduate and post-graduate levels. The quality of higher education in mathematics should be improved in such a manner that young minds are able to compete in this field globally in terms of their knowledge and skills in the globalised era of the date. Also, there is an urgent need of sustained initiatives to be taken by colleges/institutes/universities for outcome-oriented higher education in mathematics so that graduates are enabled to enhance the chances of employability. Presently, the goal of higher education in mathematics may be achieved using the following measures:

i. Curriculum reform based on a learning outcomes-based curriculum framework (LOCF).
ii. Improving learning environment and academic resources.
iii. Elevating the quality of teaching and research.

iv. Involving students in discussions, problem-solving and out of box thinking about various ideas of mathematics and their applicability, which may lead to empowerment and enhancement of the social welfare at large.
v. Encouraging the learners to make use of LOCF to learn mathematics through distance education.
vi. Motivating the learners to understand various concepts of mathematics keeping in view the regional context.
vii. Enabling learners to create research atmosphere in mathematical sciences in their colleges/institutes/universities.
viii. Teach courses of mathematics based on Choice Based Credit System (CBCS).

One of the benchmarks to measure the progress of a country is the advancement of the knowledge of mathematics. Hence, innovative measures should be taken to improve the quality of mathematical knowledge in our society. This is also because mathematics has wide ranging applications in engineering, technology and a host of other areas.

2. Learning Outcomes-based approach to Curriculum Planning

The Bachelor's Degree in B.A./B.Sc. (Hons) Mathematics and B.A./B.Sc. with Mathematics as a subject, is awarded to the students on the basis of knowledge, understanding, skills, attitudes, values and academic achievements sought to be acquired by learners at the end of these programmes. Hence, the learning outcomes of mathematics for these courses are aimed at facilitating the learners to acquire these attributes, keeping in view of their preferences and aspirations for knowledge of mathematics.

The LOCF in mathematics has designed courses in the light of graduate attributes, description of qualifications, courses and programme learning outcomes. The committee has tried to frame the syllabi of mathematics in such a way that it may lead to all round development and delivery of complete curriculum planning. Hence, it provides specific guidelines to the learners to acquire sufficient knowledge during this programme.

The objectives of LOCF (Mathematics) is to prepare the syllabi having standard level of study. It is also aimed at prescribing standard norms for teaching-learning process and

examination pattern. Hence, the programme has been chalked out in such manner that there is scope of flexibility and innovation in

i. modifications of prescribed syllabi.
ii. teaching-learning methodology.
iii. assessment technique of students and knowledge levels.
iv. learning outcomes of courses.
v. inclusion of new elective courses subject to availability of experts in colleges/institutes/universities across the country.

2.1. Nature and extent of Bachelor's Degree Programme

Mathematics is the study of quantity, structure, space and change. It has very broad scope in science, engineering and social sciences. The key areas of study in mathematics are:

1. Calculus
2. Algebra
3. Geometry
4. Differential Equations
5. Analysis
6. Mechanics

Degree programs in mathematics cover topics which are already mentioned in details under various headings in Section 6. The depth and breadth of study of individual topics depend on the nature and devotion of learners in specific mathematics programmes.

As a part of effort to enhance employability of mathematics graduates, the courses have been designed to include learning experiences, which offer them opportunities in various sectors of human activities. In this context, the experience of the project work in the areas of applications of mathematics has a key role.

2.2. Aims of Bachelor's degree programme in Mathematics

The overall aims of B.A./B.Sc. (Hons) Mathematics and B.A./B.Sc. with Mathematics as a subject are to

- create deep interest in learning mathematics.
- develop broad and balanced knowledge and understanding of definitions, concepts, principles and theorems.
- familiarize the students with suitable tools of mathematical analysis to handle issues and problems in mathematics and related sciences.
- enhance the ability of learners to apply the knowledge and skills acquired by them during the programme to solve specific theoretical and applied problems in mathematics.
- provide students/learners sufficient knowledge and skills enabling them to undertake further studies in mathematics and its allied areas on multiple disciplines concerned with mathematics.
- encourage the students to develop a range of generic skills helpful in employment, internships and social activities.

2.3. Key outcomes underpinning curriculum planning and development

The LOCF in Mathematics desires to propose the courses of mathematics for B.A./B.Sc. (Hons) Mathematics and B.A./B.Sc. with Mathematics as a subject, based on the expected learning outcomes and academic standards which are necessary for the graduates after completing these programmes. The committee considered and discussed the following factors seriously:

i. Framing of syllabi
ii. Learners attributes
iii. Qualification descriptors
iv. Programme learning outcomes
v. Course learning outcomes
vi. Necessity of having elective courses
vii. Applications of mathematics
viii. Employability in banking, finance and other sectors.

3. Graduate Attributes in Mathematics

The graduate attributes in mathematics are the summation of the expected course learning outcomes mentioned in the beginning of each course. Some of them are stated below.

3.1. Disciplinary knowledge:

Capability of demonstrating comprehensive knowledge of mathematics and understanding of one or more disciplines which form a part of an undergraduate programme of study.

3.2. Communications skills:

i. Ability to communicate various concepts of mathematics effectively using examples and their geometrical visualizations.
ii. Ability to use mathematics as a precise language of communication in other branches of human knowledge.
iii. Ability to communicate long standing unsolved problems in mathematics.
iv. Ability to show the importance of mathematics as precursor to various scientific developments since the beginning of the civilization.
v. Ability to explain the development of mathematics in the civilizational context and its role as queen of all sciences.

3.3. Critical thinking and analytical reasoning:

i. Ability to employ critical thinking in understanding the concepts in every area of mathematics.
ii. Ability to analyze the results and apply them in various problems appearing in different branches of mathematics.

3.4. Problem solving:

i. Capability to solve problems in computer graphics using concepts of linear algebra.
ii. Capability to solve various models such as growth and decay models, radioactive decay model, drug assimilation, LCR circuits and population models using techniques of differential equations.
iii. Ability to solve linear system of equations, linear programming problems and network flow problems.
iv. Ability to provide new solutions using the domain knowledge of mathematics

acquired during this programme.

3.5. Research-related skills:

i. Capability for inquiring about appropriate questions relating to the concepts in various fields of mathematics.
ii. To know about the advances in various branches of mathematics.

3.6. Information/digital literacy:

i. Capability to use appropriate softwares to solve system of equations and differential equations.
ii. Capability to understand and apply the programming concepts of C++ to mathematical investigations and problem solving.

3.7. Self-directed learning:

Ability to work independently and do in-depth study of various notions of mathematics.

3.8. Moral and ethical awareness/reasoning:

Ability to identify unethical behaviour such as fabrication, falsification or misrepresentation of data and adopting objective, unbiased and truthful actions in all aspects.

3.9. Lifelong learning:

Ability to think, acquire knowledge and skills through logical reasoning and to inculcate the habit of self-learning.

4. Qualification descriptors for B.A./B.Sc. (Hons) Mathematics and B.A./B.Sc. with Mathematics as a subject

The qualification descriptor suggests the generic outcomes and attributes to be obtained while obtaining the degree of B.A./B.Sc. (Hons) Mathematics or B.A./B.Sc. with Mathematics as a subject. The qualification descriptors indicate the academic standards on the basis of following factors:

i. Level of knowledge
ii. Understanding
iii. Skills
iv. Competencies and attitudes
v. Values.

These parameters are expected to be attained and demonstrated by the learners after becoming graduates in these programmes. The colleges/institutes/universities should consider the above mentioned parameters at the time of designing, approving, assessing and reviewing academic programmes containing common courses for B.A./B.Sc. (Hons) Mathematics as well as B.A./B.Sc. with Mathematics as a subject. The learning experiences and assessment procedures should be so designed that every graduate with mathematics may achieve the programme learning outcomes with equal opportunity irrespective of the class, gender, community and regions. Each graduate in mathematics should be able to:

i. demonstrate fundamental systematic knowledge of mathematics and its applications in engineering, science, technology and mathematical sciences. It should also enhance the subject specific knowledge and help in creating jobs in various sectors.
ii. demonstrate educational skills in areas of analysis, geometry, algebra, mechanics, differential equations etc.
iii. apply knowledge, understanding and skills to identify the difficult/unsolved problems in mathematics and to collect the required information in possible range of sources and try to analyse and evaluate these problems using appropriate methodologies.
iv. fulfil one's learning requirements in mathematics, drawing from a range of contemporary research works and their applications in diverse areas of mathematical sciences.
v. apply one's disciplinary knowledge and skills in mathematics in newer domains and uncharted areas.
vi. identify challenging problems in mathematics and obtain well-defined solutions.
vii. exhibit subject-specific transferable knowledge in mathematics relevant to job trends and employment opportunities.

5. Programme Learning Outcomes of B.A./B.Sc. (Hons) Mathematics and B.A./B.Sc. with Mathematics as a Subject

1. Bachelor's degree in mathematics is the culmination of in-depth knowledge of algebra, calculus, geometry, differential equations and several other branches of mathematics. This also leads to study of related areas like computer science and statistics. Thus, this programme helps learners in building a solid foundation for higher studies in mathematics.

2. The skills and knowledge gained has intrinsic beauty, which also leads to proficiency in analytical reasoning. This can be utilised in modelling and solving real life problems.

3. Students undergoing this programme learn to logically question assertions, to recognise patterns and to distinguish between essential and irrelevant aspects of problems. They also share ideas and insights while seeking and benefitting from knowledge and insight of others. This helps them to learn behave responsibly in a rapidly changing interdependent society.

4. Students completing this programme will be able to present mathematics clearly and precisely, make vague ideas precise by formulating them in the language of mathematics, describe mathematical ideas from multiple perspectives and explain fundamental concepts of mathematics to non-mathematicians.

5. Completion of this programme will also enable the learners to join teaching profession in primary and secondary schools.

6. This programme will also help students to enhance their employability for government jobs, jobs in banking, insurance and investment sectors, data analyst jobs and jobs in various other public and private enterprises.

6. Structure of B.A./B.Sc. Mathematics

6.1. Course learning outcomes

Course learning outcomes of each course in B.A./B.Sc. (Hons) Mathematics and B.A./B.Sc. with Mathematics as a subject have been enshrined in the beginning of course contents of each course.

B.A./B.Sc. (Hons) Mathematics

CORE COURSES (14)

Programme outcomes	Calculus	Algebra and Geometry	Multivariable Calculus	Ordinary Differential Equations	Real Analysis	Group Theory	Probability and Statistics	Mechanics	Linear Algebra	Partial Differential Equations and Calculus of Variations	Set Theory and Metric Spaces	Advanced Algebra	Complex Analysis	Numerical Analysis
Disciplinary knowledge	√	√	√	√	√	√	√	√	√	√	√	√	√	√
Communication skills	√	√	√	√	√	√	√		√	√	√	√	√	
Critical thinking	√	√	√	√	√	√	√	√	√	√	√	√	√	√
Analytical thinking	√	√	√	√	√	√	√	√	√	√	√	√	√	√
Problem solving	√	√	√	√	√	√	√	√	√	√	√	√	√	√
Research related skills				√						√		√	√	
Information literacy	√	√	√	√	√	√	√	√	√	√	√	√	√	√
Digital literacy			√				√							√
Self-directed learning	√	√	√	√	√	√	√	√	√	√	√	√	√	√
Lifelong learning	√	√	√	√	√		√			√	√	√	√	√
Professional skills	√	√	√	√	√	√	√	√	√	√	√	√	√	√
Applicational skills	√		√	√			√	√	√	√				√
Experimental learning	√	√	√	√	√		√	√	√	√			√	√
Employability options	√		√				√		√	√				√

DISCIPLINE SPECIFIC ELECTIVE COURSES (Any Four)

Programme	Tensors and Differential Gemetry	Mathematical Logic	Integral Transforms and Fourier Analysis	Linear Programming	Information Theory and Coding	Graph Theory	Special Theory and Relativity	Discrete Mathematics	Wavelets and Applications	Number Theory	Mathematical Finance	C++ Programming for Mathematics	Cryptography	Advanced Mechanics	Dissertation on Any Topic of Mathematics
Disciplinary knowledge	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√
Communication skills	√	√			√	√		√	√		√	√		√	
Critical thinking	√	√	√	√	√	√		√	√	√	√	√	√	√	√
Analytical thinking	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√
Problem solving	√	√	√	√	√	√	√	√	√	√	√		√	√	
Research related skills	√	√	√	√	√	√	√	√	√	√	√	√	√	√	
Information literacy			√	√	√				√			√			
Digital literacy			√	√	√				√			√			
Self-directed learning	√	√	√	√	√	√		√	√	√	√	√		√	√
Lifelong learning	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√
Professional skills	√	√	√	√	√	√	√	√	√	√	√	√	√	√	
Applicational skills			√	√	√	√		√	√		√	√	√		
Experimental learning				√	√	√		√	√		√	√	√		
Employability options				√	√			√	√		√	√	√		

B.A./B.Sc. with Mathematics as a Subject

CORE COURSES (4)

Programme outcomes	Calculus	Algebra	Differential Equations	Real Analysis
Disciplinary knowledge	√	√	√	√
Communication skills	√	√	√	√
Critical thinking	√	√	√	√
Analytical thinking	√	√	√	√
Problem solving	√	√	√	√
Research related skills		√	√	√
Information literacy	√	√		√
Digital literacy		√		
Self-directed learning	√	√	√	√
Lifelong learning	√		√	√
Professional skills	√	√	√	√
Applicational skills	√		√	
Experimental learning	√	√	√	
Employability options			√	

DISCIPLINE SPECIFIC ELECTIVE COURSES (Any Two)

Programme outcomes	Mechanics	Probability and Statistics	Numerical Methods	Complex Variables	Linear Algebra	Integral Transforms and Fourier Analysis	Discrete Mathematics	Linear Programming and Game Theory	Tensors and Differential Geometry	Number Theory	Advanced Mechanics	Information Theory and Coding	Special Theory of Relativity	C++ Programming for Mathematics
Disciplinary knowledge	√	√	√	√	√	√	√	√	√	√	√	√	√	√
Communication skills	√	√	√	√	√	√	√	√	√	√	√	√	√	√
Critical thinking	√	√	√	√	√	√	√	√	√	√	√	√	√	√
Analytical thinking	√		√		√	√		√	√		√	√	√	√
Problem solving	√	√	√		√	√	√	√				√	√	√
Research related skills			√			√						√	√	√
Information literacy		√	√		√	√	√	√		√		√	√	
Digital literacy		√	√		√	√	√	√		√		√	√	√
Self-directed learning	√	√	√	√	√	√	√	√	√	√	√	√	√	√
Lifelong learning	√	√	√	√	√	√	√	√	√	√	√	√	√	√
Professional skills	√	√	√	√	√	√	√	√	√	√	√	√	√	√
Applicational skills		√	√		√	√	√	√		√		√	√	√
Experimental learning		√	√		√	√	√	√				√	√	
Employability options	√	√	√	√	√	√	√	√	√	√	√	√	√	√

6.1.1. Credit distribution for B.A./B.Sc. (Hons) Mathematics

Sl. No.	Nature of Papers	Total No. of Papers	Credits in Theory+(Tutorial/Practical)	Total Credits
1.	Core Papers	14	06	84
2.	DSE (Discipline Specific Electives) Papers	04	06	24
3.	Generic Electives /Interdisciplinary	04	06	24
4.	Ability Enhancement Papers	02	04	08
5.	Skill Enhancement Papers	02	04	08
Total Papers/Credits		28	--	148

6.1.2. Credit distribution for B.A./B.Sc. with Mathematics as a subject

S. No.	Nature of Papers	Total No. of Papers	Credits in Theory + Tutorial	Total Credits
1.	Core Papers	04	06	24
2.	DSE (Discipline Specific Elective) Papers	02	06	12
3.	Skill Enhancement Papers	02	04	08
Total Papers/Credits		06		44

6.2. Contents for each course

6.2.1. Contents of courses for B.A./B.Sc. (Hons) Mathematics

Semesters	**Core Courses**	**DSE Courses**
I	**Paper-101:** Calculus **Paper-102:** Algebra and Geometry	
II	**Paper-201:** Multivariable Calculus **Paper-202:** Ordinary Differential Equations	
III	**Paper-301:** Real Analysis **Paper-302:** Group Theory **Paper-303:** Probability and	

	Statistics	
IV	**Paper-401:**Mechanics **Paper-402:** Linear Algebra **Paper-403:** Partial Differential Equations and Calculus of Variations	
V	**Paper-501:** Set Theory and Metric Spaces **Paper-502:** Advanced Algebra	**(Any Two)** **Paper-503 & 504 (i)-(vii)** **Paper-(i):** Tensors and Differential Geometry **Paper-(ii):** Mathematical Logic **Paper-(iii):** Integral Transforms and Fourier Analysis **Paper-(iv):** Linear Programming **Paper-(v):** Information Theory and Coding **Paper-(vi):** Graph Theory **Paper-(vii):** Special Theory and Relativity
VI	**Paper-601:** Complex Analysis **Paper-602:** Numerical Analysis	**(Any Two)** **Paper-603 &604 (i)-(viii)** **Paper-(i):** Discrete Mathematics **Paper-(ii):** Wavelets and Applications **Paper-(iii):** Number Theory **Paper-(iv):** Mathematical Finance **Paper-(v):** C++Programming for Mathematics **Paper-(vi):** Cryptography **Paper-(vii):** Advanced Mechanics **Paper-(viii):** Dissertation on Any Topic of Mathematics

Semester-I

Paper-101: Calculus

Course Learning Outcomes: This course will enable the students to:

i) Assimilate the notions of limit of a sequence and convergence of a series of real numbers.
ii) Calculate the limit and examine the continuity of a function at a point.
iii) Understand the consequences of various mean value theorems for differentiable functions.
iv) Sketch curves in Cartesian and polar coordinate systems.
v) Apply derivative tests in optimization problems appearing in social sciences, physical sciences, life sciences and a host of other disciplines.

Unit-I: Sequences and Integration

Real numbers, Sequences of real numbers, Convergence of sequences and series, Bounded and monotonic sequences; Definite integral as a limit of sum, Integration of irrational algebraic functions and transcendental functions, Reduction formulae, Definite integrals.

Unit-II: Limit and Continuity

ε-δ definition of limit of a real valued function, Limit at infinity and infinite limits; Continuity of a real valued function, Properties of continuous functions, Intermediate value theorem, Geometrical interpretation of continuity, Types of discontinuity; Uniform continuity.

Unit-III: Differentiability

Differentiability of a real valued function, Geometrical interpretation of differentiability, Relation between differentiability and continuity, Differentiability and monotonicity, Chain rule of differentiation; Darboux's theorem, Rolle's theorem, Lagrange's mean value theorem, Cauchy's mean value theorem, Geometrical interpretation of mean value theorems; Successive differentiation, Leibnitz's theorem.

Unit-IV: Expansions of Functions

Maclaurin's and Taylor's theorems for expansion of a function in an infinite series, Taylor's theorem in finite form with Lagrange, Cauchy and Roche–Schlomilch forms of remainder; Maxima and minima.

Unit-V: Curvature, Asymptotes and Curve Tracing

Curvature; Asymptotes of general algebraic curves, Parallel asymptotes, Asymptotes parallel to axes; Symmetry, Concavity and convexity, Points of inflection, Tangents at origin, Multiple points, Position and nature of double points; Tracing of Cartesian, polar and parametric curves.

References:

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2. Gabriel Klambauer (1986). *Aspects of Calculus*. Springer-Verlag.
3. Wieslaw Krawcewicz & Bindhyachal Rai (2003). *Calculus with Maple Labs*. Narosa.
4. Gorakh Prasad (2016). *Differential Calculus* (19th edition). Pothishala Pvt. Ltd.
5. George B. Thomas Jr., Joel Hass, Christopher Heil & Maurice D. Weir (2018). *Thomas' Calculus* (14th edition). Pearson Education.

Paper-102: Algebra and Geometry

Course Learning Outcomes: This course will enable the students to:

i) Understand the importance of roots of real and complex polynomials and learn various methods of obtaining roots.
ii) Familiarize with relations, equivalence relations and partitions.
iii) Employ De Moivre's theorem in a number of applications to solve numerical problems.
iv) Recognize consistent and inconsistent systems of linear equations by the row echelon form of the augmented matrix, using rank.
v) Find eigenvalues and corresponding eigenvectors for a square matrix.
vi) Explain the properties of three dimensional shapes.

Unit-I: Theory of Equations and Complex Numbers

Elementary theorems on the roots of an equations including Cardan's method, The remainder and factor theorems, Synthetic division, Factored form of a polynomial, The Fundamental theorem of algebra, Relations between the roots and the coefficients of polynomial equations, Imaginary roots, Integral and rational roots; Polar representation of complex numbers, The n^{th} roots of unity, De Moivre's theorem for integer and rational indices and its applications.

Unit-II: Relations and Basic Number Theory

Relations, Equivalence relations, Equivalence classes; Functions, Composition of functions, Inverse of a function; Finite, countable and uncountable sets; The division algorithm, Divisibility and the Euclidean algorithm, The fundamental theorem of arithmetic, Modular arithmetic and basic properties of congruences; Principles of mathematical induction and well ordering.

Unit-III: Row Echelon Form of Matrices and Applications

Systems of linear equations, Row reduction and echelon forms, Linear independence, The rank of a matrix and applications; Introduction to linear transformations, The matrix of a linear transformation, Matrix operations, Determinants, The inverse of a matrix, Characterizations of invertible matrices; Applications to Computer Graphics; Eigenvalues and eigenvectors, The characteristic equation and the Cayley–Hamilton theorem.

Unit-IV: Planes, Straight Lines and Spheres

Planes: Distance of a point from a plane, Angle between two planes, pair of planes, Bisectors of angles between two planes; Straight lines: Equations of straight lines, Distance of a point from a straight line, Distance between two straight lines, Distance between a straight line and a plane; Spheres: Different forms, Intersection of two spheres, Orthogonal intersection, Tangents and normal, Radical plane, Radical line, Coaxial system of spheres, Pole, Polar and Conjugacy.

Unit-V: Locus, Surfaces, Curves and Conicoids

Space curves, Algebraic curves, Ruled surfaces, Some standard surfaces, Classification of quadric surfaces, Cone, Cylinder, Central conicoids, Tangent plane, Normal, Polar planes, and Polar lines.

References:

1. Titu Andreescu, & Dorin Andrica (2014). *Complex Numbers from A to...Z*. (2nd edition). Birkhäuser.
2. Robert J. T. Bell (1994). *An Elementary Treatise on Coordinate Geometry of Three Dimensions*. Macmillan India Ltd.
3. D. Chatterjee (2009). *Analytical Geometry: Two and Three Dimensions*. Narosa Publishing House.
4. Leonard Eugene Dickson (2009). *First Course in the Theory of Equations*. The Project Gutenberg EBook (http://www.gutenberg.org/ebooks/29785)
5. Edgar G. Goodaire & Michael M. Parmenter (2015). *Discrete Mathematics with Graph Theory* (3rd edition). Pearson Education Pvt. Ltd. India.
6. Bernard Kolman & David R. Hill (2003). *Introductory Linear Algebra with Applications* (7th edition). Pearson Education Pvt. Ltd. India.
7. David C. Lay, Steven R. Lay & Judi J. McDonald (2016). *Linear Algebra and its Applications* (5th edition). Pearson Education Pvt. Ltd. India.

Semester-II

Paper-201: Multivariable Calculus

Course Learning Outcomes: This course will enable the students to:

i) Learn conceptual variations while advancing from one variable to several variables in calculus.
ii) Apply multivariable calculus in optimization problems.
iii) Inter-relationship amongst the line integral, double and triple integral formulations.
iv) Applications of multivariable calculus tools in physics, economics, optimization, and understanding the architecture of curves and surfaces in plane and space etc.
v) Realize importance of Green, Gauss and Stokes' theorems in other branches of mathematics.

Unit-I: Partial Differentiation

Functions of several variables, Level curves and surfaces, Limits and continuity, Partial differentiation, Tangent planes, Chain rule, Directional derivatives, The gradient, Maximal and normal properties of the gradient, Tangent planes and normal lines.

Unit-II: Differentiation

Higher order partial derivatives, Total differential and differentiability, Jacobians, Change of variables, Euler's theorem for homogeneous functions, Taylor's theorem for functions of two variables and more variables, Envelopes and evolutes.

Unit-III: Extrema of Functions and Vector Field

Extrema of functions of two and more variables, Method of Lagrange multipliers, Constrained optimization problems, Definition of vector field, Divergence, curl, gradient and vector identities.

Unit-IV: Double and Triple Integrals

Double integration over rectangular and nonrectangular regions, Double integrals in polar co-ordinates, Triple integral over a parallelepiped and solid regions, Volume by triple integrals, Triple integration in cylindrical and spherical coordinates, Change of variables in double and triple integrals, Dirichlet integral.

Unit-V: Green's, Stokes' and Gauss Divergence Theorem

Line integrals, Applications of line integrals**:** Mass and Work, Fundamental theorem for line integrals, Conservative vector fields, Green's theorem, Area as a line integral, Surface integrals, Stokes' theorem, The Gauss divergence theorem.

References:

1. Jerrold Marsden, Anthony J. Tromba & Alan Weinstein (2009). *Basic Multivariable Calculus*, Springer India Pvt. Limited.
2. James Stewart (2012). *Multivariable Calculus* (7th edition). Brooks/Cole. Cengage.
3. Monty J. Strauss, Gerald L. Bradley & Karl J. Smith (2011). *Calculus* (3rd edition). Pearson Education. Dorling Kindersley (India) Pvt. Ltd.
4. George B. Thomas Jr., Joel Hass, Christopher Heil & Maurice D. Weir (2018). *Thomas' Calculus* (14th edition). Pearson Education.

Paper-202: Ordinary Differential Equations

Course Learning Outcomes: The course will enable the students to:

i) Understand the genesis of ordinary differential equations.

ii) Learn various techniques of getting exact solutions of solvable first order differential equations and linear differential equations of higher order.

iii) Know Picard's method of obtaining successive approximations of solutions of first order differential equations, passing through a given point in the plane and Power series method for higher order linear equations, especially in cases when there is no method available to solve such equations.

iv) Grasp the concept of a general solution of a linear differential equation of an arbitrary order and also learn a few methods to obtain the general solution of such equations.

v) Formulate mathematical models in the form of ordinary differential equations to suggest possible solutions of the day to day problems arising in physical, chemical and biological disciplines.

Unit-I: First Order Differential Equations

Basic concepts and genesis of ordinary differential equations, Order and degree of a differential equation, Differential equations of first order and first degree, Equations in which variables are separable, Homogeneous equations, Linear differential equations and equations reducible to linear form, Exact differential equations, Integrating factor, First order higher degree equations solvable for x, y and p. Clairaut's form and singular solutions. Picard's method of successive approximations and the statement of Picard's theorem for the existence and uniqueness of the solutions of the first order differential equations.

Unit-II: Second Order Linear Differential Equations

Statement of existence and uniqueness theorem for linear differential equations, General theory of linear differential equations of second order with variable coefficients, Solutions of homogeneous linear ordinary differential equations of second order with constant coefficients, Transformations of the equation by changing the dependent/independent variable, Method of variation of parameters and method of undetermined coefficients, Reduction of order, Coupled linear differential equations with constant coefficients.

Unit-III: Higher Order Linear Differential Equations

Principle of superposition for a homogeneous linear differential equation, Linearly dependent and linearly independent solutions on an interval, Wronskian and its properties, Concept of a general solution of a linear differential equation, Linear homogeneous and non-homogeneous equations of higher order with constant coefficients, Euler-Cauchy equation, Method of variation of parameters and method of undetermined coefficients, Inverse operator method.

Unit-IV: Series Solutions of Differential Equations

Power series method, Legendre's equation, Legendre polynomials, Rodrigue's formula, Orthogonality of Legendre polynomials, Frobenius method, Bessel's equation, Bessel functions and their properties, Recurrence relations.

Unit-V: Applications

Orthogonal trajectories, Acceleration-velocity model, Minimum velocity of escape from Earth's gravitational field, Growth and decay models, Malthusian and logistic population models, Radioactive decay, Drug assimilation into the blood of a single cold pill; Free and forced mechanical oscillations of a spring suspended vertically carrying a mass at its lowest tip, Phenomena of resonance, LCR circuits, Lotka–Volterra population model.

References:

1. Belinda Barnes & Glenn Robert Fulford (2015). *Mathematical Modelling with Case Studies*: *A Differential Equation Approach Using* Maple *and* MATLAB (2nd edition). Chapman & Hall/CRC Press, Taylor & Francis.
2. H. I. Freedman (1980). *Deterministic Mathematical Models in Population Ecology*. Marcel Dekker Inc.
3. Erwin Kreyszig (2011). *Advanced Engineering Mathematics* (10th edition). Wiley.
4. Daniel A. Murray (2003). *Introductory Course in Differential Equations*, Orient.
5. B. Rai, D. P. Choudhury & H. I. Freedman (2013). *A Course in Ordinary Differential Equations* (2nd edition). Narosa.
6. Shepley L. Ross (2007). *Differential Equations* (3rd edition), Wiley India.
7. George F. Simmons (2017). *Differential Equations with Applications and Historical Notes* (3rd edition). CRC Press. Taylor & Francis.

Semester-III

Paper-301: Real Analysis

Course Learning Outcomes: This course will enable the students to:

i) Understand many properties of the real line $\mathbb{R}$ and learn to define sequence in terms of functions from $\mathbb{R}$ to a subset of $\mathbb{R}$.

ii) Recognize bounded, convergent, divergent, Cauchy and monotonic sequences and to calculate their limit superior, limit inferior, and the limit of a bounded sequence.

iii) Apply the ratio, root, alternating series and limit comparison tests for convergence and absolute convergence of an infinite series of real numbers.

iv) Learn some of the properties of Riemann integrable functions, and the applications of the fundamental theorems of integration.

Unit-I: Real Number System

Algebraic and order properties of $\mathbb{R}$, Absolute value of a real number; Bounded above and bounded below sets, Supremum and infimum of a nonempty subset of $\mathbb{R}$,The completeness property of $\mathbb{R}$, Archimedean property, Density of rational numbers in $\mathbb{R}$, Definition and types of intervals, Nested intervals property; Neighborhood of a point in $\mathbb{R}$, Open, closed and perfect sets in $\mathbb{R}$, Connected subsets of $\mathbb{R}$, Cantor set and Cantor function.

Unit-II: Sequences of Real Numbers

Convergent sequence, Limit of a sequence, Bounded sequence, Limit theorems, Monotone sequences, Monotone convergence theorem, Subsequences, Bolzano−Weierstrass theorem for sequences, Limit superior and limit inferior of a sequence of real numbers, Cauchy sequence, Cauchy's convergence criterion.

Unit-III: Infinite Series

Convergence and divergence of infinite series of positive real numbers, Necessary condition for convergence, Cauchy criterion for convergence; Tests for convergence of positive term series**;** Basic comparison test, Limit comparison test, D'Alembert's ratio test, Cauchy's n^{th} root test, Integral test; Alternating series, Leibniz test, Absolute and conditional convergence, Rearrangement of series and Riemann's theorem.

Unit-IV: Riemann Integration

Riemann integral, Integrability of continuous and monotonic functions, Fundamental theorem of integral calculus, First mean value theorem, Bonnet and Weierstrass forms of second mean value theorems.

Unit-V: Uniform convergence and Improper integral:

Pointwise and uniform convergence of sequence and series of functions, Weierstrass's M-test, Dirichlet test and Abel's test for uniform convergence, Uniform convergence and continuity, Uniform convergence and differentiability, Improper integrals, Dirichlet test and Abel's test for improper integrals.

References:

1. Robert G. Bartle & Donald R. Sherbert (2015). *Introduction to Real Analysis* (4th edition).Wiley India.
2. Gerald G. Bilodeau, Paul R. Thie & G. E. Keough (2015). *An Introduction to Analysis* (2nd edition), Jones and Bartlett India Pvt. Ltd.
3. K. A. Ross (2013). *Elementary Analysis***:** *The Theory of Calculus* (2nd edition). Springer.

Paper-302: Group Theory

Course Learning Outcomes: The course will enable the students to:

i) Recognize the mathematical objects called groups.

ii) Link the fundamental concepts of groups and symmetries of geometrical objects.

iii) Explain the significance of the notions of cosets, normal subgroups, and factor groups.

iv) Analyze consequences of Lagrange's theorem.

v) Learn about structure preserving maps between groups and their consequences.

Unit-I: Groups and its Elementary Properties

Symmetries of a square, Definition and examples of groups including dihedral, permutation and quaternion groups, Elementary properties of groups.

Unit-II: Subgroups and Cyclic Groups

Subgroups and examples of subgroups, Cyclic groups, Properties of cyclic groups, Lagrange's theorem, Euler phi function, Euler's theorem, Fermat's little theorem.

Unit-III: Normal Subgroups

Properties of cosets, Normal subgroups, Simple groups, Factor groups, Cauchy's theorem for finite abelian groups; Centralizer, Normalizer, Center of a group, Product of two subgroups; Classification of subgroups of cyclic groups.

Unit-IV: Permutation Groups

Cycle notation for permutations, Properties of permutations, Even and odd permutations, alternating groups, Cayley's theorem and its applications.

Unit-V: Group Homomorphisms, Rings and Fields

Group homomorphisms, Properties of homomorphisms, Group isomorphisms, Properties of isomorphisms; First, second and third isomorphism theorems for groups; Definitions and elementary properties of rings and fields.

References:

1. Michael Artin (2014). *Algebra* (2nd edition). Pearson.
2. John B. Fraleigh (2007). *A First Course in Abstract Algebra* (7th edition). Pearson.
3. Joseph A. Gallian (2017). *Contemporary Abstract Algebra* (9th edition). Cengage.
4. I. N. Herstein (2006). *Topics in Algebra* (2nd edition). Wiley India.
5. Nathan Jacobson (2009). *Basic Algebra* I (2nd edition). Dover Publications.

6. Ramji Lal (2017). *Algebra 1: Groups, Rings, Fields and Arithmetic*. Springer.
7. I.S. Luthar & I.B.S. Passi (2013). *Algebra: Volume 1: Groups*. Narosa.

Paper-303: Probability and Statistics

Course Learning Outcomes: This course will enable the students to:

i) Understand distributions in the study of the joint behaviour of two random variables.
ii) Establish a formulation helping to predict one variable in terms of the other that is, correlation and linear regression.
iii) Understand central limit theorem, which establish the remarkable fact that the empirical frequencies of so many natural populations, exhibit a bell shaped curve.

Unit-I: Probability Functions and Moment Generating Function

Basic notions of probability, Conditional probability and independence, Baye's theorem; Random variables - Discrete and continuous, Cumulative distribution function, Probability mass/density functions; Transformations, Mathematical expectation, Moments, Moment generating function, Characteristic function.

Unit-II: Univariate Discrete and Continuous Distributions

Discrete distributions**:** Uniform, Bernoulli, Binomial, Negative binomial, Geometric and Poisson; Continuous distributions**:** Uniform, Gamma, Exponential, Chi-square, Beta and normal; Normal approximation to the binomial distribution.

Unit-III: Bivariate Distribution

Joint cumulative distribution function and its properties, Joint probability density function, Marginal distributions, Expectation of function of two random variables, Joint moment generating function, Conditional distributions and expectations.

Unit-IV: Correlation, Regression and Central Limit Theorem

The Correlation coefficient, Covariance, Calculation of covariance from joint moment generating function, Independent random variables, Linear regression for two variables, The method of least squares, Bivariate normal distribution, Chebyshev's theorem, Strong law of large numbers, Central limit theorem and weak law of large numbers.

Unit-V: Modeling Uncertainty

Uncertainty, Information and entropy, Uniform Priors, Polya's urn model and random graphs.

References:

1. Robert V. Hogg, Joseph W. McKean & Allen T. Craig (2013). *Introduction to Mathematical Statistics* (7th edition), Pearson Education.

2. Irwin Miller &Marylees Miller (2014). *John E. Freund's Mathematical Statistics with Applications* (8th edition). Pearson. Dorling Kindersley Pvt. Ltd. India.
3. Jim Pitman (1993). *Probability*, Springer-Verlag.
4. Sheldon M. Ross (2014). *Introduction to Probability Models* (11th edition). Elsevier.
5. A. M. Yaglom and I. M. Yaglom (1983). *Probability and Information*. D. Reidel Publishing Company. Distributed by Hindustan Publishing Corporation (India) Delhi.

Semester-IV

Paper-401: Mechanics

Course Learning Outcomes: This course will enable the students to:

i) Familiarize with subject matter, which has been the single centre, to which were drawn mathematicians, physicists, astronomers, and engineers together.

ii) Understand necessary conditions for the equilibrium of particles acted upon by various forces and learn the principle of virtual work for a system of coplanar forces acting on a rigid body.

iii) Determine the centre of gravity of some materialistic systems and discuss the equilibrium of a uniform cable hanging freely under its own weight.

iv) Deal with the kinematics and kinetics of the rectilinear and planar motions of a particle including the constrained oscillatory motions of particles.

v) Learn that a particle moving under a central force describes a plane curve and know the Kepler's laws of the planetary motions, which were deduced by him long before the mathematical theory given by Newton.

Unit-I: Statics

Equilibrium of a particle, Equilibrium of a system of particles, Necessary conditions of equilibrium, Moment of a force about a point, Moment of a force about a line, Couples, Moment of a couple, Equipollent system of forces, Work and potential energy, Principle of virtual work for a system of coplanar forces acting on a particle or at different points of a rigid body, Forces which can be omitted in forming the equations of virtual work.

Unit-II: Centres of Gravity and Common Catenary

Centres of gravity of plane area including a uniform thin straight rod, triangle, circular arc, semicircular area and quadrant of a circle, Centre of gravity of a plane area bounded by a curve, Centre of gravity of a volume of revolution; Flexible strings, Common catenary, Intrinsic and Cartesian equations of the common catenary, Approximations of the catenary.

Unit-III: Rectilinear Motion

Simple harmonic motion (SHM) and its geometrical representation, SHM under elastic forces, Motion under inverse square law, Motion in resisting media, Concept of terminal velocity, Motion of varying mass.

Unit-IV: Motion in a Plane

Kinematics and kinetics of the motion, Expressions for velocity and acceleration in Cartesian, polar and intrinsic coordinates; Motion in a vertical circle, projectiles in a vertical plane and cycloidal motion.

Unit-V: Central Orbits

Equation of motion under a central force, Differential equation of the orbit, (p, r) equation of the orbit, Apses and apsidal distances, Areal velocity, Characteristics of central orbits, Kepler's laws of planetary motion

References:

1. S. L. Loney (2006). *An Elementary Treatise on the Dynamics of a Particle and of Rigid Bodies*. Read Books.
2. P. L. Srivatava (1964). *Elementary Dynamics*. Ram Narin Lal, Beni Prasad Publishers Allahabad.
3. J. L. Synge & B. A. Griffith (1949). *Principles of Mechanics*. McGraw-Hill.
4. A. S. Ramsey (2009). *Statics*. Cambridge University Press.
5. A. S. Ramsey (2009). *Dynamics*. Cambridge University Press.
6. R. S. Varma (1962). *A Text Book of Statics*. Pothishala Pvt. Ltd.

Paper-402: Linear Algebra

Course Learning Outcomes: This course will enable the students to:

i) Understand the concepts of vector spaces, subspaces, bases, dimension and their properties.
ii) Relate matrices and linear transformations, compute eigen values and eigen vectors of linear transformations.
iii) Learn properties of inner product spaces and determine orthogonality in inner product spaces.
iv) Realise importance of adjoint of a linear transformation and its canonical form.

Unit-I: Vector Spaces

Definition and examples, Subspace, Linear span, Quotient space and direct sum of subspaces, Linearly independent and dependent sets, Bases and dimension.

Unit-II: Linear Transformations

Definition and examples, Algebra of linear transformations, Matrix of a linear transformation, Change of coordinates, Rank and nullity of a linear transformation and rank-nullity theorem.

Unit-III: Further Properties of Linear Transformations

Isomorphism of vector spaces, Isomorphism theorems, Dual and second dual of a vector space, Transpose of a linear transformation, Eigen vectors and eigen values of a linear transformation, Characteristic polynomial and Cayley–Hamilton theorem, Minimal polynomial.

Unit-IV: Inner Product Spaces

Inner product spaces and orthogonality, Cauchy–Schwarz inequality, Gram–Schmidt orthogonalisation, Diagonalisation of symmetric matrices.

Unit-V: Adjoint of a Linear Transformation and Canonical Forms

Adjoint of a linear operator; Hermitian, unitary and normal linear transformations; Jordan canonical form, Triangular form, Trace and transpose, Invariant subspaces.

References:

1. Stephen H. Friedberg, Arnold J. Insel & Lawrence E. Spence (2003). *Linear Algebra* (4th edition). Prentice-Hall of India Pvt. Ltd.
2. Kenneth Hoffman & Ray Kunze (2015). *Linear Algebra* (2nd edition). Prentice-Hall.
3. I. M. Gel'fand (1989). *Lectures on Linear Algebra.* Dover Publications.

4. Nathan Jacobson (2009). *Basic Algebra* I & II (2nd edition). Dover Publications.
5. Serge Lang (2005). *Introduction to Linear Algebra* (2nd edition). Springer India.
6. Vivek Sahai & Vikas Bist (2013). *Linear Algebra* (2nd Edition). Narosa Publishing House.
7. Gilbert Strang (2014). *Linear Algebra and its Applications* (2nd edition). Elsevier.

Paper-403: Partial Differential Equations and Calculus of Variations

Course Learning Outcomes: This course will enable the students to:

i) Apply a range of techniques to solve first & second order partial differential equations.
ii) Model physical phenomena using partial differential equations such as the heat and wave equations.
iii) Understand problems, methods and techniques of calculus of variations.

Unit-I: First Order Partial Differential Equations

Order and degree of Partial differential equations (PDE), Concept of linear and non-linear partial differential equations, Partial differential equations of the first order, Lagrange's method, Some special type of equation which can be solved easily by methods other than the general method, Charpit's general method.

Unit-II: Second Order Partial Differential Equations with Constant Coefficients

Classification of linear partial differential equations of second order, Homogeneous and non-homogeneous equations with constant coefficients.

Unit-III: Second Order Partial Differential Equations with Variable Coefficients

Partial differential equations reducible to equations with constant coefficient, Second order PDE with variable coefficients, Classification of second order PDE, Reduction to canonical or normal form; Monge's method; Solution of heat and wave equations in one and two dimensions by method of separation of variables.

Unit-IV: Calculus of Variations-Variational Problems with Fixed Boundaries

Euler's equation for functional containing first order and higher order total derivatives, Functionals containing first order partial derivatives, Variational problems in parametric form, Invariance of Euler's equation under coordinates transformation.

Unit-V: Calculus of Variations-Variational Problems with Moving Boundaries

Variational problems with moving boundaries, Functionals dependent on one and two variables, One sided variations. Sufficient conditions for an extremum-Jacobi and Legendre conditions, Second variation.

References:

1. A. S. Gupta (2004). *Calculus of Variations with Applications*. PHI Learning.

2. Erwin Kreyszig (2011). *Advanced Engineering Mathematics* (10th edition). Wiley.
3. TynMyint-U & Lokenath Debnath (2013). *Linear Partial Differential Equation for Scientists and Engineers* (4th edition). Springer India.
4. H. T. H. Piaggio (2004). An Elementary Treatise on *Differential Equations and Their Applications.* CBS Publishers.
5. S. B. Rao & H. R. Anuradha (1996). *Differential Equations with Applications*. University Press.
6. Ian N. Sneddon (2006). *Elements of Partial Differential Equations*. Dover Publications.

Semester-V

Paper-501: Set Theory and Metric Spaces

Course Learning Outcomes: This course will enable the students to:

i) Learn basic facts about the cardinality of a set.
ii) Understand several standard concepts of metric spaces and their properties like openness, closedness, completeness, Bolzano–Weierstrass property, compactness, and connectedness.
iii) Identify the continuity of a function defined on metric spaces and homeomorphisms.

Unit-I: Theory of Sets

Finite and infinite sets, Countable and uncountable sets, Cardinality of sets, Schröder–Bernstein theorem, Cantor's theorem, Order relation in cardinal numbers, Arithmetic of cardinal numbers, Partially ordered set, Zorn's lemma and Axiom of choice, Various set theoretic paradoxes.

Unit-II: Concepts in Metric Spaces

Definition and examples of metric spaces, Open spheres and closed spheres, Neighbourhoods, Open sets, Interior, exterior and boundary points, Closed sets, Limit points and isolated points, Interior and closure of a set, Boundary of a set, Bounded sets, Distance between two sets, Diameter of a set, Subspace of a metric space.

Unit-III: Complete Metric Spaces and Continuous Functions

Cauchy and Convergent sequences, Completeness of metric spaces, Cantor's intersection theorem, Dense sets and separable spaces, Nowhere dense sets and Baire's category theorem, Continuous and uniformly continuous functions, Homeomorphism, Banach contraction principle.

Unit-IV: Compactness

Compact spaces, Sequential compactness, Bolzano–Weierstrass property, Compactness and finite intersection property, Heine–Borel theorem, Totally bounded sets, Equivalence of compactness and sequential compactness, Continuous functions on compact spaces.

Unit-V: Connectedness

Separated sets, Disconnected and connected sets, Components, Connected subsets of $\mathbb{R}$, Continuous functions on connected sets.

References:

1. E. T. Copson (1988). *Metric Spaces*. Cambridge University Press.
2. P. R. Halmos (1974). *Naive Set Theory*. Springer.
3. P. K. Jain & Khalil Ahmad (2019). *Metric Spaces*. Narosa.
4. S. Kumaresan (2011). *Topology of Metric Spaces* (2nd edition). Narosa.
5. Satish Shirali & Harikishan L. Vasudeva (2006). *Metric Spaces*. Springer-Verlag.
6. Micheál O'Searcoid (2009). *Metric Spaces*. Springer-Verlag.
7. G. F. Simmons (2004). *Introduction to Topology and Modern Analysis*. McGraw-Hill.

Paper-502: Advanced Algebra

Course Learning Outcomes: This course will enable the students to:

i) Understand the basic concepts of group actions and their applications.

ii) Recognize and use the Sylow theorems to characterize certain finite groups.

iii) Know the fundamental concepts in ring theory such as the concepts of ideals, quotient rings, integral domains, and fields.

iv) Learn in detail about polynomial rings, fundamental properties of finite field extensions, and classification of finite fields.

Unit-I: Group Actions

Group actions, Orbits and stabilizers, Conjugacy classes, Orbit-stabilizer theorem, Normalizer of an element of a group, Center of a group, Class equation of a group, Inner and outer automorphisms of a group.

Unit-II: Sylow Theorems

Cauchy's theorem for finite abelian groups, Finite simple groups, Sylow theorems and applications including nonsimplicity tests.

Unit-III: Rings and Fields

Definition, examples and elementary properties of rings, Commutative rings, Integral domain, Division rings and fields, Characteristic of a ring, Ring homomorphisms and isomorphisms, Ideals and quotient rings. Prime, principal and maximal ideals, Relation between integral domain and field, Euclidean rings and their properties, Wilson and Fermat's theorems.

Unit-IV: Polynomial Rings

Polynomial rings over commutative ring and their basic properties, The division algorithm; Polynomial rings over rational field, Gauss lemma and Eisenstein's criterion, Euclidean domain, principal ideal domain, and unique factorization domain.

Unit-V: Field Extensions and Finite Fields

Extension of a field, Algebraic element of a field, Algebraic and transcendental numbers, Perfect field, Classification of finite fields.

References:

1. Michael Artin (2014). *Algebra* (2nd edition). Pearson.

2. P. B. Bhattacharya, S. K. Jain & S. R. Nagpaul (2003). *Basic Abstract Algebra* (2nd edition). Cambridge University Press.
3. David S. Dummit & Richard M. Foote (2008). *Abstract Algebra* (2nd edition). Wiley.
4. John B. Fraleigh (2007). *A First Course in Abstract Algebra* (7th edition). Pearson.
5. Joseph A. Gallian (2017). *Contemporary Abstract Algebra* (9th edition). Cengage.
6. N. S. Gopalakrishnan (1986). *University Algebra*, New Age International Publishers.
7. I. N. Herstein (2006). *Topics in Algebra* (2nd edition). Wiley India.
8. Thomas W. Hungerford (2004). *Algebra* (8th edition). Springer.
9. Nathan Jacobson (2009). *Basic Algebra I & II* (2nd edition). Dover Publications.
10. Serge Lang (2002). *Algebra* (3rd edition). Springer-Verlag.
11. I. S. Luthar & I. B. S. Passi (2013). *Algebra: Volume 1: Groups*. Narosa.
12. I. S. Luthar & I. B. S. Passi (2012). *Algebra: Volume 2: Rings*. Narosa.

Elective Courses (Any Two)

(Paper-503 & 504 (i)-(vii))

Paper-(i):Tensors and Differential Geometry

Course Learning Outcomes: This course will enable the students to:

i) Explain the basic concepts of tensors.

ii) Understand role of tensors in differential geometry.

iii) Learn various properties of curves including Frenet–Serret formulae and their applications.

iv) Know the Interpretation of the curvature tensor, Geodesic curvature, Gauss and Weingarten formulae.

v) Understand the role of Gauss's Theorema Egregium and its consequences.

vi) Apply problem-solving with differential geometry to diverse situations in physics, engineering and in other mathematical contexts.

Unit-I: Tensors

Contravariant and covariant vectors, Transformation formulae, Tensor product of two vector spaces, Tensor of type (r, s), Symmetric and skew-symmetric properties, Contraction of tensors, Quotient law, Inner product of vectors.

Unit-II: Further Properties of Tensors

Fundamental tensors, Associated covariant and contravariant vectors, Inclination of two vectors and orthogonal vectors, Christoffel symbols, Law of transformation of Christoffel symbols, Covariant derivatives of covariant and contravariant vectors, Covariant differentiation of tensors, Curvature tensor, Ricci tensor, Curvature tensor identities.

Unit-III: Curves in $\mathbb{R}^2$ and $\mathbb{R}^3$

Basic definitions and examples, Arc length, Curvature and the Frenet–Serret formulae, Fundamental existence and uniqueness theorem for curves, Non-unit speed curves.

Unit-IV: Surfaces in $\mathbb{R}^3$

Basic definitions and examples, The first fundamental form, Arc length of curves on surfaces, Normal curvature, Geodesic curvature, Gauss and Weingarten formulae, Geodesics, Parallel vector fields along a curve and parallelism.

Unit-V: Geometry of Surfaces

The second fundamental form and the Weingarten map; Principal, Gauss and mean curvatures; Isometries of surfaces, Gauss's Theorema Egregium, The fundamental theorem of surfaces, Surfaces of constant Gauss curvature, Exponential map, Gauss lemma, Geodesic coordinates, The Gauss–Bonnet formula and theorem.

References:

1. Christian Bär (2010). *Elementary Differential Geometry*. Cambridge University Press.
2. Manfredo P. do Carmo (2016). *Differential Geometry of Curves & Surfaces* (Revised and updated 2nd edition). Dover Publications.
3. Alferd Gray (2018). *Modern Differential Geometry of Curves and Surfaces with Mathematica* (4th edition). Chapman & Hall/CRC Press, Taylor & Francis.
4. Richard S. Millman & George D. Parkar (1977). *Elements of Differential Geometry*. Prentice-Hall.
5. R. S. Mishra (1965). *A Course in Tensors with Applications to Riemannian Geometry*. Pothishala Pvt. Ltd.
6. Sebastián Montiel & Antonio Ross (2009). *Curves and Surfaces*. American Mathematical Society.

Paper-(ii): Mathematical Logic

Course Learning Outcomes: This course will enable the students to:

i) Learn the syntax of first-order logic and semantics of first-order languages.

ii) Understand the propositional logic and basic theorems like compactness theorem, meta theorem and post-tautology theorem.

iii) Assimilate the concept of completeness interpretations and their applications with special emphasis on applications in algebra.

Unit-I: Syntax of First-order Logic

First-order languages, Terms of language, Formulas of language, First order theory.

Unit-II: Semantics of First-order Languages

Structures of first order languages, Truth in a structure, Model of a theory, Embeddings and isomorphism.

Unit-III: Propositional Logics

Syntax of propositional logic, Semantics of propositional logic, Compactness theorem for propositional logic, Proof in propositional logic, Meta theorem in propositional logic, Post tautology theorem.

Unit-IV: Proof and Meta Theorems in First-order Logic

Proof in first-order logic, Meta theorems in first-order logic, Some meta theorem in arithmetic, Consistency and completeness.

Unit-V: Completeness Theorem and Model Theory

Completeness theorem, Interpretation in a theory, Extension by definitions, Compactness theorem and applications, Complete theories, Applications in algebra.

References:

1. Richard E. Hodel (2013). *An Introduction to Mathematical Logic.* Dover Publications.
2. Yu I. Manin (2010). *A Course in Mathematical Logic for Mathematicians* (2^{nd} edition). Springer.
3. Elliott Mendelson (2015). *Introduction to Mathematical Logic* (6^{th} edition). Chapman & Hall/CRC.
4. Shashi Mohan Srivastava (2013). *A Course on Mathematical Logic* (2^{nd} edition). Springer.

Paper-(iii): Integral Transforms and Fourier Analysis

Course Learning Outcomes: This course will enable the students to:

i) Know about piecewise continuous functions, Dirac delta function, Laplace transforms and its properties.
ii) Solve ordinary differential equations using Laplace transforms.
iii) Familiarise with Fourier transforms of functions belonging to $L^1(\mathbb{R})$ class, relation between Laplace and Fourier transforms.
iv) Explain Parseval's identity, Plancherel's theorem and applications of Fourier transforms to boundary value problems.
v) Learn Fourier series, Bessel's inequality, term by term differentiation and integration of Fourier series.
vi) Apply the concepts of the course in real life problems.

Unit-I: Laplace Transforms

Laplace transform, Linearity, Existence theorem, Laplace transforms of derivatives and integrals, Shifting theorems, Change of scale property, Laplace transforms of periodic functions, Dirac's delta function.

Unit-II: Further Properties of Laplace Transforms and Applications

Differentiation and integration of transforms, Convolution theorem, Integral equations, Inverse Laplace transform, Lerch's theorem, Linearity property of inverse Laplace transform, Translations theorems of inverse Laplace transform, Inverse transform of derivatives, Applications of Laplace transform in obtaining solutions of ordinary differential equations and integral equations.

Unit-III: Fourier Transforms

Fourier and inverse Fourier transforms, Fourier sine and cosine transforms, Inverse Fourier sine and cosine transforms, Linearity property, Change of scale property, Shifting property, Modulation theorem, Relation between Fourier and Laplace transforms.

Unit-IV: Solution of Equations by Fourier Transforms

Solution of integral equation by Fourier sine and cosine transforms, Convolution theorem for Fourier transform, Parseval's identity for Fourier transform, Plancherel's theorem, Fourier transform of derivatives, Applications of infinite Fourier transforms to boundary value problems, Finite Fourier transform, Inversion formula for finite Fourier transforms.

Unit-V: Fourier Series

Fourier cosine and sine series, Fourier series, Differentiation and integration of Fourier series, Absolute and uniform convergence of Fourier series, Bessel's inequality, The complex form of Fourier series.

References:

1. James Ward Brown & Ruel V. Churchill (2011). F*ourier Series and Boundary Value Problems*. McGraw-Hill Education.
2. Charles K. Chui (1992). *An Introduction to Wavelets*. Academic Press.
3. Erwin Kreyszig (2011). *Advanced Engineering Mathematics* (10th edition). Wiley.
4. Walter Rudin (2017). *Fourier Analysis on Groups*. Dover Publications.
5. A. Zygmund (2002).*Trigonometric Series* (3rd edition). Cambridge University Press.

Paper-(iv): Linear Programming

Course Learning Outcomes: This course will enable the students to:

i) Analyze and solve linear programming models of real life situations.
ii) Provide graphical solutions of linear programming problems with two variables, and illustrate the concept of convex set and extreme points.
iii) Understand the theory of the simplex method.
iv) Know about the relationships between the primal and dual problems, and to understand sensitivity analysis.
v) Learn about the applications to transportation, assignment and two-person zero-sum game problems.

Unit-I: Linear Programming Problem, Convexity and Basic Feasible Solutions

Formulation, Canonical and standard forms, Graphical method; Convex and polyhedral sets, Hyperplanes, Extreme points; Basic solutions, Basic Feasible Solutions, Reduction of feasible solution to basic feasible solution, Correspondence between basic feasible solutions and extreme points.

Unit-II: Simplex Method

Optimality criterion, Improving a basic feasible solution, Unboundedness, Unique and alternate optimal solutions; Simplex algorithm and its tableau format; Artificial variables, Two-phase method, Big-M method.

Unit-III: Duality

Formulation of the dual problem, Duality theorems, Complimentary slackness theorem, Economic interpretation of the dual, Dual-simplex method.

Unit-IV: Sensitivity Analysis

Changes in the cost vector, right-hand side vector and the constraint matrix of the linear programming problem.

Unit-V: Applications

Transportation Problem: Definition and formulation, Methods of finding initial basic feasible solutions: Northwest-corner rule, Least- cost method, Vogel approximation method; Algorithm for obtaining optimal solution.

Assignment Problem: Mathematical formulation and Hungarian method.

Game Theory: Formulation and solution of two-person zero-sum games, Games with mixed strategies, Linear programming method for solving a game.

References:

1. Mokhtar S. Bazaraa, John J. Jarvis & Hanif D. Sherali (2010). *Linear Programming and Network Flows* (4th edition). John Wiley & Sons.
2. G. Hadley (2002). *Linear Programming*. Narosa Publishing House.
3. Frederick S. Hillier & Gerald J. Lieberman (2015). *Introduction to Operations Research* (10th edition). McGraw-Hill Education.
4. Hamdy A. Taha (2017). *Operations Research*: *An Introduction* (10th edition). Pearson.
5. Paul R. Thie & Gerard E. Keough (2014). *An Introduction to Linear Programming and Game Theory* (3rd edition). Wiley India Pvt. Ltd.

Paper-(v): Information Theory and Coding

Course Learning Outcomes: This course will enable the students to:

i) Study simple ideal statistical communication models.

ii) Understand the development of codes for transmission and detection of information.

iii) Learn about the input and output of a signal via transmission channel.

iv) Study detection and correction of errors during transmission.

v) Represent a linear code by matrices - encoding and decoding.

Unit-I: Concepts of Information Theory

Communication processes, A model of communication system, A quantitative measure of information, Binary unit of information, A measure of uncertainty, H function as a measure of uncertainty, Sources and binary sources, Measure of information for two-dimensional discrete finite probability schemes.

Unit-II: Entropy Function

A sketch of communication network, Entropy, Basic relationship among different entropies, A measure of mutual information, Interpretation of Shannon's fundamental inequalities; Redundancy, efficiency, and channel capacity; Binary symmetric channel, Binary erasure channel, Uniqueness of the entropy function, Joint entropy and conditional entropy, Relative entropy and mutual information, Chain rules for entropy, Conditional relative entropy and conditional mutual information, Jensen's inequality and its characterizations, The log sum inequality and its applications.

Unit-III: Concepts of Coding

Block codes, Hamming distance, Maximum likelihood decoding, Levels of error handling, Error correction, Error detection, Erasure correction, Construction of finite fields, Linear codes, Matrix representation of linear codes, Hamming codes.

Unit-IV: Bounds of Codes

Orthogonality relation, Encoding and decoding of linear codes, The singleton bound and maximum distance separable codes, The sphere-packing bound and perfect codes, The Gilbert–Varshamov bound, MacWilliams' identities.

Unit-V: Cyclic Codes

Definition and examples of cyclic codes, Generator polynomial and check polynomial, Generator matrix and check matrix, Bose–Chaudhuri–Hocquenghem (BCH) code as a cyclic code.

References:

1. Robert B. Ash, (2014). *Information Theory*. Dover Publications.
2. Thomas M. Cover & Joy A. Thomas (2013). *Elements of Information Theory* (2nd edition). Wiley India Pvt. Ltd.
3. Joseph A. Gallian (2017). *Contemporary Abstract Algebra* (9th edition), Cengage.
4. Fazlollah M. Reza, (2003). *An Introduction to Information Theory*. Dover Publications.
5. Ron M. Roth (2007). *Introduction to Coding Theory*. Cambridge University Press.
6. Claude E. Shannon & Warren Weaver (1969). *The Mathematical Theory of Communication*. The University of Illinios Press.

Paper-(vi): Graph Theory

Course Learning Outcomes: This course will enable the students to:

i) Appreciate the definition and basics of graphs along with types and their examples.
ii) Understand the definition of a tree and learn its applications to fundamental circuits.
iii) Know the applications of graph theory to network flows.
iv) Understand the notion of planarity and coloring of a graph.
v) Relate the graph theory to the real-world problems.

Unit-I: Paths, Circuits and Graph Isomorphisms

Definition and examples of a graph, Subgraph, Walks, Paths and circuits; Connected graphs, disconnected graphs and components of a graph; Euler and Hamiltonian graphs, Graph isomorphisms, Adjacency matrix and incidence matrix of a graph, Directed graphs and their elementary properties.

Unit-II: Trees and Fundamental Circuits

Definition and properties of trees, Rooted and binary trees, Cayley's theorem on a counting tree, Spanning tree, Fundamental circuits, Minimal spanning trees in a connected graph.

Unit-III: Cut-Sets and Cut-Vertices

Cut-set of a graph and its properties, Fundamental circuits and cut-sets, Cut-vertices, Connectivity and separability, Network flows, 1- isomorphism and 2- isomorphism.

Unit-IV: Planar Graphs

Planar graph, Euler theorem for a planar graph, Various representations of a planar graph, Dual of a planar graph, Detection of planarity, Kuratowski's theorem.

Unit-V: Graph Coloring

Chromatic number of a graph, Chromatic partition, Chromatic polynomial, Matching and coverings, Four color problem.

References:

1. R. Balakrishnan & K. Ranganathan (2012). *A Textbook of Graph Theory*. Springer.
2. Narsingh Deo (2016). *Graph Theory with Applications to Engineering and Computer Science*. Dover Publications.
3. Reinhard Diestel (2017). *Graph Theory* (5th edition). Springer.
4. Edgar G. Goodaire & Michael M. Parmenter (2018). *Discrete Mathematics with Graph Theory* (3rd edition). Pearson.
5. Douglas West (2017). *Introduction to Graph Theory* (2nd edition). Pearson.

Paper-(vii): Special Theory of Relativity

Course Learning Outcomes: This course will enable the students to:

i) Understand the basic elements of Newtonian mechanics including Michelson–Morley experiment and geometrical interpretations of Lorentz transformation equations.

ii) Learn about length contraction, time dilation and Lorentz contraction factor.

iii) Study 4-dimensional Minkowskian space-time and its consequences.

iv) Understand equations of motion as a part of relativistic mechanics.

v) Imbibe connections between relativistic mechanics and electromagnetism.

Unit-I: Newtonian Mechanics

Inertial frames, Speed of light and Gallilean relativity, Michelson–Morley experiment, Lorentz–Fitzgerold contraction hypothesis, Relative character of space and time, Postulates of special theory of relativity, Lorentz transformation equations and its geometrical interpretation, Group properties of Lorentz transformations.

Unit-II: Relativistic Kinematics

Composition of parallel velocities, Length contraction, Time dilation, Transformation equations for components of velocity and acceleration of a particle and Lorentz contraction factor.

Unit-III: Geometrical representation of space-time

Four dimensional Minkowskian space-time of special relativity, Time-like, light-like and space-like intervals, Null cone, Proper time, World line of a particle, Four vectors and tensors in Minkowiskian space-time.

Unit-IV: Relativistic Mechanics

Variation of mass with velocity. Equivalence of mass and energy. Transformation equations for mass momentum and energy. Energy-momentum four vector. Relativistic force and Transformation equations for its components. Relativistic equations of motion of a particle.

Unit-V: Electromagnetism

Transformation equations for the densities of electric charge and current. Transformation equations for electric and magnetic field strengths. The Field of a Uniformly Moving Point charge. Forces and fields near a current carrying wire. Forces between moving charges. The invariance of Maxwell`s equations.

References:

1. James L. Anderson (1973). *Principles of Relativity Physics*. Academic Press.
2. Peter Gabriel Bergmann (1976). *Introduction to the Theory of Relativity*. Dover Publications.
3. C. Moller (1972). *The Theory of Relativity* (2nd edition). Oxford University Press.
4. Robert Resnick (2007). *Introduction to Special Relativity*. Wiley.
5. Wolfgang Rindler (1977). *Essential Relativity: Special, General, and Cosmological*. Springer-Verlag.
6. V. A. Ugarov (1979). *Special Theory of Relativity*. Mir Publishers, Moscow.

Semester-VI

Paper-601: Complex Analysis

Course Learning Outcomes: This course will enable the students to:

i) Visualize complex numbers as points of $\mathbb{R}^2$ and stereographic projection of complex plane on the Riemann sphere.

ii) Understand the significance of differentiability and analyticity of complex functions leading to the Cauchy–Riemann equations.

iii) Learn the role of Cauchy–Goursat theorem and Cauchy integral formula in evaluation of contour integrals.

iv) Apply Liouville's theorem in fundamental theorem of algebra.

v) Understand the convergence, term by term integration and differentiation of a power series.

vi) Learn Taylor and Laurent series expansions of analytic functions, classify the nature of singularity, poles and residues and application of Cauchy Residue theorem.

Unit-I: Complex Plane and functions.

Complex numbers and their representation, algebra of complex numbers; Complex plane, Open set, Domain and region in complex plane; Stereographic projection and Riemann sphere; Complex functions and their limits including limit at infinity; Continuity, Linear fractional transformations and their geometrical properties.

Unit-II: Analytic Functions and Cauchy–Riemann Equations

Differentiability of a complex valued function, Cauchy–Riemann equations, Harmonic functions, necessary and sufficient conditions for differentiability, Analytic functions; Analyticity and zeros of exponential, trigonometric and logarithmic functions; Branch cut and branch of multi-valued functions.

Unit-III: Cauchy's Theorems and Fundamental Theorem of Algebra

Line integral, Path independence, Complex integration, Green's theorem, Anti-derivative theorem, Cauchy–Goursat theorem, Cauchy integral formula, Cauchy's inequality, Derivative of analytic function, Liouville's theorem, Fundamental theorem of algebra, Maximum modulus theorem and its consequences.

Unit-IV: Power Series

Sequences, series and their convergence, Taylor series and Laurent series of analytic functions, Power series, Radius of convergence, Integration and differentiation of power series, Absolute and uniform convergence of power series.

Unit-V: Singularities and Contour Integration

Meromorphic functions, Zeros and poles of meromorphic functions, Nature of singularities, Picard's theorem, Residues, Cauchy's residue theorem, Argument principle, Rouche's theorem, Jordan's lemma, Evaluation of proper and improper integrals.

References:

1. Lars V. Ahlfors (2017). *Complex Analysis* (3rd edition). McGraw-Hill Education.
2. Joseph Bak & Donald J. Newman (2010). *Complex Analysis* (3rd edition). Springer.
3. James Ward Brown & Ruel V. Churchill (2009). *Complex Variables and Applications* (9th edition). McGraw-Hill Education.
4. John B. Conway (1973). *Functions of One Complex Variable*. Springer-Verlag.
5. E.T. Copson (1970). *Introduction to Theory of Functions of Complex Variable*. Oxford University Press.
6. Theodore W. Gamelin (2001). *Complex Analysis*. Springer-Verlag.
7. George Polya & Gordon Latta (1974). *Complex Variables*. Wiley.
8. H. A. Priestley (2003). *Introduction to Complex Analysis*. Oxford University Press.
9. E. C. Titchmarsh (1976). *Theory of Functions* (2nd edition). Oxford University Press.

Paper-602: Numerical Analysis

Course Learning Outcomes: This course will enable the students to:

i) Obtain numerical solutions of algebraic and transcendental equations.

ii) Find numerical solutions of system of linear equations and check the accuracy of the solutions.

iii) Learn about various interpolating and extrapolating methods.

iv) Solve initial and boundary value problems in differential equations using numerical methods.

v) Apply various numerical methods in real life problems.

Unit-I: Numerical Methods for Solving Algebraic and Transcendental Equations

Round-off error and computer arithmetic, Local and global truncation errors, Algorithms and convergence; Bisection method, False position method, Fixed point iteration method, Newton's method and secant method for solving equations.

Unit-II: Numerical Methods for Solving Linear Systems

Partial and scaled partial pivoting, Lower and upper triangular (LU) decomposition of a matrix and its applications, Thomas method for tridiagonal systems; Gauss–Jacobi, Gauss–Seidel and successive over-relaxation (SOR) methods.

Unit-III: Interpolation

Lagrange and Newton interpolations, Piecewise linear interpolation, Cubic spline interpolation, Finite difference operators, Gregory–Newton forward and backward difference interpolations.

Unit-IV: Numerical Differentiation and Integration

First order and higher order approximation for first derivative, Approximation for second derivative; Numerical integration: Trapezoidal rule, Simpson's rules and error analysis, Bulirsch–Stoer extrapolation methods, Richardson extrapolation.

Unit-V: Initial and Boundary Value Problems of Differential Equations

Euler’s method, Runge–Kutta methods, Higher order one step method, Multi-step methods; Finite difference method, Shooting method, Real life examples: Google search engine, 1D and 2D simulations, Weather forecasting.

References:

1. Brian Bradie (2006), *A Friendly Introduction to Numerical Analysis.* Pearson.
2. C. F. Gerald & P. O. Wheatley (2008). *Applied Numerical Analysis* (7th edition), Pearson Education, India.
3. F. B. Hildebrand (2013). *Introduction to Numerical Analysis*: (2nd edition). Dover Publications.
4. M. K. Jain, S. R. K. Iyengar & R. K. Jain (2012). *Numerical Methods for Scientific and Engineering Computation* (6th edition). New Age International Publishers.
5. Robert J. Schilling & Sandra L. Harris (1999). *Applied Numerical Methods for Engineers Using* MATLAB *and C*. Thomson-Brooks/Cole.

Elective Courses (Any two)

(Paper-603 &604 (i)-(vii)

Paper-(i): Discrete Mathematics

Course Learning Outcomes: This course will enable the students to:

i) Learn about partially ordered sets, lattices and their types.

ii) Understand Boolean algebra and Boolean functions, logic gates,switching circuitsand their applications.

iii) Solve real-life problems using finite-state and Turing machines.

iv) Assimilate various graph theoretic concepts and familiarize with their applications.

Unit-I: Partially Ordered Sets

Definitions, examples and basic properties of partially ordered sets (poset), Order isomorphism, Hasse diagrams, Dual of a poset, Duality principle, Maximal and minimal elements, Least upper bound and greatest upper bound, Building new poset, Maps between posets.

Unit-II: Lattices

Lattices as posets, Lattices as algebraic structures, Sublattices, Products and homomorphisms; Definitions, examples and properties of modular and distributive lattices; Complemented, relatively complemented and sectionally complemented lattices.

Unit-III:Boolean Algebras and Switching Circuits

Boolean algebras, De Morgan's laws, Boolean homomorphism, Representation theorem; Boolean polynomials, Boolean polynomial functions, Disjunctive and conjunctive normal forms, Minimal forms of Boolean polynomials, Quine–McCluskey method, Karnaugh diagrams, Switching circuits and applications.

Unit-IV: Finite-State and Turing Machines

Finite-state machines with outputs, and with no output; Deterministic and nodeterministic finite-state automaton; Turing machines: Definition, examples, and computations.

Unit-V: Graphs

Definition, examples and basic properties of graphs, Königsberg bridge problem; Subgraphs, Pseudographs, Complete graphs, Bipartite graphs, Isomorphism of graphs, Paths and circuits, Eulerian circuits, Hamiltonian cycles, Adjacency matrix, Weighted graph, Travelling-salesman problem, Shortest path and Dijkstra's algorithm.

References:

1. B. A. Davey & H. A. Priestley (2002). *Introduction to Lattices and Order* (2nd edition). Cambridge University Press.
2. Edgar G. Goodaire & Michael M. Parmenter (2018). *Discrete Mathematics with Graph Theory* (3rd edition). Pearson Education.
3. Rudolf Lidl & Günter Pilz (1998). *Applied Abstract Algebra* (2nd edition). Springer.
4. Kenneth H. Rosen (2012). *Discrete Mathematics and its Applications*: *With Combinatorics and Graph Theory* (7th edition). McGraw-Hill.
5. C. L. Liu (1985). *Elements of Discrete Mathematics* (2nd edition). McGraw-Hill.

Paper-(ii): Wavelets and Applications

Course Learning Outcomes: This course will enable the students to:

i) Know basic concepts of signals and systems.

ii) Understand the concept of Haar spaces.

iii) Learn Fourier transform and wavelet transform of digital signals.

iv) Learn applications of wavelets to the real-world problems.

v) Apply wavelets in signal processing and image processing.

Unit-I: Signals and Systems

Basic concepts of signals and systems, Frequency spectrum of signals; Classification of signals: Discrete time signals and continuous time signals, periodic and non-periodic signals; Classification of systems: Linear, nonlinear, time-variant, time-invariant, stable and unstable systems.

Unit-II: Haar Scaling Function and Wavelet, Time-Frequency Analysis

Orthogonal functions, Orthonormal functions, Function spaces, Orthogonal basis functions, Haar scaling function, Haar spaces: Haar space V_0, general Haar space V_j; Haar wavelet, Haar wavelet spaces: Haar wavelet space W_0, general Haar wavelet space W_j; Decomposition and reconstruction, Time-frequency analysis, Orthogonal and orthonormal bases.

Unit–III: Fourier Transforms and Wavelets

Discrete Fourier transform of a digital signal, Complex form of a Fourier series, Inverse discrete Fourier transform, Window Fourier transform, Short time Fourier transform, Admissibility condition for a wavelet, Classes of wavelets: Haar, Morlet, Maxican hat, Meyer and Daubechies wavelets; Wavelets with compact support.

Unit–IV: Discrete Wavelet Transforms

Stationary and non-stationary signals, Haar transform, 1-level Haar transform, Multi-level Haar transform, Conservation and compaction of energy, Multiresolution analysis, Decomposition and reconstruction of signals using discrete wavelet transform (DWT).

Unit–V: Applications

Wavelet series expansion using Haar and other wavelets, Applications in signal compression, Analysis and classification of audio signals using DWT, Signal de-noising: Image and ECG signals.

References:

1. Charles K. Chui (1992). *An Introduction to Wavelets*. Academic Press.
2. Ingrid Daubechies (1999). *Ten Lectures on Wavelets*. SIAM
3. Michael W. Frazier (1999). *An Introduction to Wavelets Through Linear Algebra*. Springer-Verlag.
4. Stéphane Mallat (2008). *A Wavelet Tour of Signal Processing* (3rd edition). Academic Press.
5. M.J. Roberts (2004). *Signals and Systems: Analysis Using Transform Methods and* MATLAB. McGraw-Hill Education.
6. David K. Ruch & Patrick J. Van Fleet (2009), *Wavelet Theory*: *An Elementary Approach with Applications*. John Wiley & Sons.
7. James S. Walker (2008). *A Primer on Wavelets and Their Scientific Applications* (2nd edition). Chapman & Hall/CRC, Taylor & Francis.

Paper-(iii): Number Theory

Course Learning Outcomes: This course will enable the students to:

i) Learn about some important results in the theory of numbers including the prime number theorem, Chinese remainder theorem, Wilson's theorem and their consequences.
ii) Learn about number theoretic functions, modular arithmetic and their applications.
iii) Familiarise with modular arithmetic and find primitive roots of prime and composite numbers.
iv) Know about open problems in number theory, namely, the Goldbach conjecture and twin-prime conjecture.
v) Apply public crypto systems, in particular, RSA.

Unit-I: Distribution of Primes and Theory of Congruencies

Linear Diophantine equation, Prime counting function, Prime number theorem, Goldbach conjecture, Twin-prime conjecture, Odd perfect numbers conjecture, Fermat and Mersenne primes, Congruence relation and its properties, Linear congruence and Chinese remainder theorem, Fermat's little theorem, Wilson's theorem.

Unit-II: Number Theoretic Functions

Number theoretic functions for sum and number of divisors, Multiplicative function, The Möbius inversion formula, Greatest integer function, Euler's phi-function and properties, Euler's theorem.

Unit-III: Primitive Roots

Order of an integer modulo n, Primitive roots for primes, Composite numbers having primitive roots; Definition of quadratic residue of an odd prime, Euler's criterion.

Unit-IV: Quadratic Reciprocity Law

The Legendre symbol and its properties, Quadratic reciprocity, Quadratic congruencies with composite moduli.

Unit-V: Applications

Public key encryption, RSA encryption and decryption with applications in security systems.

References:

1. David M. Burton (2007). *Elementary Number Theory* (7th edition). McGraw-Hill.

2. Gareth A. Jones & J. Mary Jones (2005). *Elementary Number Theo*ry. Springer.
3. Neville Robbins (2007). *Beginning Number Theory* (2nd edition). Narosa.
4. I.Niven (2012). *An Introduction to the Theory of Numbers* (5th edition). John Wiley & Sons.
5. Neal Koblitz (1994). *A Course in Number Theory and Cryptography* (2nd edition). Springer-Verlag.

Paper-(iv): Mathematical Finance

Course Learning Outcomes: This course will enable the students to:

i) Understand financial markets and derivatives including options and futures.

ii) Appreciate pricing and hedging of options, interest rate swaps and no-arbitrage pricing concepts.

iii) Learn stochastic analysis, Ito's formula, Ito integral and the Black−Scholes model.

iv) Study and use Hedging parameters, trading strategies and currency swaps.

Unit-I: Basic Theory of Interest and Fixed-Income Securities

Principal and interest: simple, compound and continuous; Present and future value of cash flow streams; Net present value, Internal rates of return and their comparison; Inflation, Annuities; Bonds, Bond prices and yields, Macaulay duration and modified duration.

Unit-II: Term Structure of Interest Rates, Bonds and Derivatives

Spot rates, forward rates and explanations of term structure; Running present value, Floating-rate bonds, Immunization, Convexity; Putable and callable bonds; Exchange-traded markets and over-the-counter markets; Derivatives: Forward contracts, Future contracts, Options, Types of traders, Hedging, Speculation, Arbitrage.

Unit-III: Mechanics of Options Markets

No-arbitrage principle, Short selling, Forward price for an investment asset; Types of options: Call and put options, Option positions, Underlying assets, Factors affecting option prices, Upper and lower bounds for option prices, Put-call parity, Effect of dividends.

Unit-IV: Stochastic Analysis of Stock Prices and Black–Scholes Model

Binomial option pricing model, Risk neutral valuation: European and American options on assets following binomial tree model; Lognormal property of stock prices, Distribution of rate of return, Expected return, Volatility, Estimating volatility from historical data, Extension of risk-neutral valuation to assets following geometric Brownian motion, Black−Scholes formula for European options.

Unit-V: Hedging Parameters, Trading Strategies and Swaps

Hedging parameters: Delta, gamma, theta, rho and vega; Trading strategies involving options, Swaps, Mechanics of interest rate swaps, Comparative advantage argument, Valuation of interest rate swaps, Currency swaps, Valuation of currency swaps.

References:

1. John C. Hull & Sankarshan Basu (2018). *Options, Futures and Other Derivatives* (10th edition). Pearson Education.
2. David G. Luenberger (2013). *Investment Science* (2nd edition). Oxford University Press.
3. Sheldon M. Ross (2011). *An Elementary Introduction to Mathematical Finance* (3rd edition). Cambridge University Press.

Paper-(v): C++Programming for Mathematics

Course Learning Outcomes: This course will enable the students to:

i) Understand and apply the programming concepts of C++ which is important for mathematical investigation and problem solving.
ii) Use mathematical libraries for computational objectives.
iii) Represent the outputs of programs visually in terms of well formatted text and plots.

Unit-I: C++ Essentials

Fundamentals of programming, Organization of logic flow in stored program model of computation, C++ as a general purpose programming language, Structure of a C++ program, Common compilers and IDE's, Basic data-types, Variables and literals in C++, Operators, Expressions, Evaluation precedence and type compatibility; Outline of program development in C++, Debugging and testing; Applications**:** Greatest common divisor and random number generation.

Unit-II: Structured Data

Structured data-types in C++, Arrays and manipulating data in arrays; Objects and classes**:** Information hiding, modularity, constructors and destructors, methods and polymorphism; Applications**:** Factorization of an integer, Euler's totient, Images in Cartesian geometry using points in two & three dimensions, Pythagorean triples.

Unit-III: Containers and Templates

Containers and Template Libraries**:** Sets, iterators, multisets, vectors, maps, lists, stacks and queues; Applications**:** Basic set algebra, modulo arithmetic and congruences, projective plane, permutations, monotone sequences and polynomials.

Unit-IV: Libraries and Packages

Libraries and Packages for arbitrary precision arithmetic and linear algebra**;** Features of C++ for input/output and visualization**:** Strings, streams, formatting methods, processing files in a batch, command-line arguments, visualization packages and their uses; Applications**:** Arbitrary precision arithmetic using GMP, BOOST; Finding nullity, rank, eigen values, eigen vectors, linear transformations, systems of linear equations; Plots.

Unit-V: Odds and Ends

Runtime errors and graceful degradation, Robustness in a program; Exception handling: Try-catch and throw; Defining and deploying suitable exception handlers in programs; Compiler

options; Conditional compilation; Understanding and defining suitable pragmas; Applications**:** Identification and description of install parameters of mathematical libraries, debugging installation, working with multiple libraries simultaneously and maintaining correctness and consistency of data.

References:

1. Nell Dale & Chip Weems (2013).*Programming and Problem Solving with C++* (6th edition). Jones & Bartlett Learning.
2. Peter Gottschling (2016). *Discovering Modern C++*: *An Intensive Course for Scientists, Engineers, and Programmers*. Pearson.
3. Nicolai M. Josuttis (2012). *The C++ Standard Library*: *A Tutorial and Reference* (2nd edition). Addison-Wesley, Pearson.
4. Donald E. Knuth (1968). *The Art of Computer Programming.* Addison-Wesley.
5. Edward Scheinerman (2006). *C++ for Mathematicians*: *An Introduction for Students and Professionals*. Chapman & Hall/CRC. Taylor & Francis.
6. B. Stroustrup (2013). *The C++ Programming Language* (4th edition). Addison-Wesley.

Paper-(vi): Cryptography

Course Learning Outcomes: This course will enable the students to:

i) Understand the difference between classical and modern cryptography.

ii) Learn the fundamentals of cryptography, including Data and Advanced Encryption Standards (DES & AES) and RSA.

iii) Encrypt and decrypt messages using block ciphers, sign and verify messages using well-known signature generation and verification algorithms.

iv) Know about the aspects of number theory which are relevant to cryptography.

Unit I: Introduction to Cryptography and Classical Cryptography

Cryptosystems and basic cryptographic tools: Secret-key cryptosystems, Public-key cryptosystems, Block and stream ciphers, Hybrid cryptography, Message integrity: Message authentication codes, Signature schemes, Nonrepudiation, Certificates, Hash functions, Cryptographic protocols, Security; Hybrid cryptography: Message integrity, Cryptographic protocols, Security, Some simple cryptosystems, Shift cipher, Substitution cipher, Affine cipher, Vigenère cipher, Hill cipher, Permutation cipher, Stream ciphers, Cryptanalysis of affine, substitution, Vigenère, Hill and LFSR stream ciphers.

Unit-II: Cryptographic Security, Pseudo Randomness and Symmetric Key Ciphers

Shannon's theory, Perfect secrecy, Entropy, Spurious keys and unicity distance; Bit generators, Security of pseudorandom bit generators. Substitution-permutation networks, Data encryption standard (DES), Description and analysis of DES; Advanced encryption standard (AES), Description and analysis of AES; Stream ciphers, Trivium.

Unit-III: Basics of Number Theory and Public-Key Cryptography

Basics of number theory; Introduction to public-key cryptography, RSA cryptosystem, Implementing RSA; Primality testing, Legendre and Jacobi symbols, Solovay–Strassen algorithm, Miller–Rabin algorithm; Square roots modulo n, Factoring algorithms, Pollard $p-1$ algorithm, Pollard rho algorithm, Dixon's random squares algorithm, Factoring algorithms in practice; Rabin cryptosystem and its security.

Unit-IV: More on Public-Key Cryptography

Basics of finite fields; ElGamal cryptosystem, Algorithms for the discrete logarithm problem, Shanks' algorithm, Pollard rho discrete logarithm algorithm, Pohlig–Hellman

algorithm; Discrete logarithm algorithms in practice, Security of ElGamal systems, Bit security of discrete logarithms.

Unit-V: Hash Functions and Signature Schemes

Hash functions and data integrity, SHA-3; RSA signature scheme, Security requirements for signature schemes, Signatures and Hash functions, ElGamal signature scheme, Security of ElGamal signature scheme, Certificates.

References:

1. Jeffrey Hoffstein, Jill Pipher & Joseph H. Silverman (2014). *An Introduction to Mathematical Cryptography* (2^{nd} edition). Springer.
2. Neal Koblitz (1994). *A Course in Number Theory and Cryptography* (2^{nd} edition). Springer-Verlag.
3. Christof Paar & Jan Pelzl (2014). *Understanding Cryptography*. Springer.
4. Simon Rubinstein-Salzedo (2018). C*ryptography*. Springer.
5. Douglas R. Stinson & Maura B. Paterson (2019). *Cryptography Theory and Practice* (4th edition). Chapman & Hall/CRC Press, Taylor & Francis.

Paper-(vii): Advanced Mechanics

Course Learning Outcomes: This course will enable the students to:

i) Understand the reduction of force system in three dimensions to a resultant force acting at a base point and a resultant couple, which is independent of the choice of base of reduction.

ii) Learn about a nul point, a nul line, and a nul plane with respect to a system of forces acting on a rigid body together with the idea of central axis.

iii) Know the inertia constants for a rigid body and the equation of momental ellipsoid together with the idea of principal axes and principal moments of inertia and to derive Euler's equations of motion of a rigid body, moving about a point which is kept fixed.

iv) Study the kinematics and kinetics of fluid motions to understand the equation of continuity in Cartesian, cylindrical polar and spherical polar coordinates which are used to derive Euler's equations and Bernoulli's equation.

v) Deal with two-dimensional fluid motion using the complex potential and also to understand the concepts of sources, sinks, doublets and the image systems of these with regard to a line and a circle.

Unit-I: Statics in Space

Forces in three dimensions, Reduction to a force and a couple, Equilibrium of a system of particles, Central axis and Wrench, Equation of the central axis, Resultant wrench of two wrenches; Nul points, lines and planes with respect to a system of forces, Conjugate forces and conjugate lines.

Unit-II: Motion of a Rigid Body

Moments and products of inertia of some standard bodies, Momental ellipsoid, Principal axes and moments of inertia; Motion of a rigid body with a fixed point, Kinetic energy of a rigid body with a fixed point and angular momentum of a rigid body, Euler's equations of motion for a rigid body with a fixed point, Velocity and acceleration of a moving particle in cylindrical and spherical polar coordinates, Motion about a fixed axis, Compound pendulum.

Unit-III: Kinematics of Fluid Motion

Lagrangian and Eulerian approaches, Material and convective derivatives, Velocity of a fluid at a point, Equation of continuity in Cartesian, cylindrical polar and spherical polar coordinates, Cylindrical and spherical symmetry, Boundary surface, Streamlines and

pathlines, Steady and unsteady flows, Velocity potential, Rotational and irrotational motion, Vorticity vector and vortex lines.

Unit-IV: Kinetics of Fluid Motion

Euler's equations of motion in Cartesian, cylindrical polar and spherical polar coordinates; Bernoulli's equation, Impulsive motion.

Unit-V: Motion in Two-Dimensions

Stream function, Complex potential, Basic singularities: Sources, sinks, doublets, complex potential due to these basic singularities; Image system of a simple source and a simple doublet with regard to a line and a circle, Milne−Thomson circle theorem.

References:

1. A. S. Ramsay (1960). *A Treatise on Hydromechanics, Part-II Hydrodynamics.* G. Bell & Sons.
2. F. Chorlton (1967). *A Textbook of Fluid Dynamics*. CBS Publishers.
3. Michel Rieutord (2015). *Fluid Dynamics An Introduction.* Springer.
4. E. A. Milne (1965). *Vectorial Mechanics*, Methuen & Co.Limited. London.

Paper-(viii): Dissertation on Any Topic of Mathematics

In this course, students are encouraged to choose the topic of their interest and do an in-depth study of the same and with some illuminating real time applications under supervision of a faculty member.

6.2.2. Contents of courses for B.A./B.Sc. with Mathematics as a subject

Semesters	Core Courses	DSE Courses
I	**Paper-M101:** Calculus	
II	**Paper-M201:** Algebra	
III	**Paper-M301:** Differential Equations	
IV	**Paper- M401:**Real Analysis	
V		(Any One) **Paper-M501(i)-(vi)** **Paper-(i):** Mechanics **Paper-(ii):** Probability and Statistics **Paper-(iii):** Numerical Methods **Paper- (iv):** Complex Variables **Paper-(v):** Linear Algebra **Paper-(vi):** Integral Transforms and Fourier Analysis
VI		(Any One) **Paper-M601(i)-(vii)** **Paper-(i):** Discrete Mathematics **Paper-(ii):** Linear Programming and Game Theory **Paper-(iii):** Tensors and Differential Geometry **Paper-(iv):** Number Theory **Paper-(v):** Advanced Mechanics **Paper-(vi):** Information Theory and Coding **Paper-(vii):** Special Theory of Relativity **Paper-(viii):** C++ Programming for Mathematics

Semester-I

Paper-M101: Calculus

Course Learning Outcomes: This course will enable the students to:

i) Calculate the limit and examine the continuity and understand the geometrical interpretation of differentiability.

ii) Understand the consequences of various mean value theorems.

iii) Draw curves in Cartesian and polar coordinate systems.

iv) Understand conceptual variations while advancing from one variable to several variables in calculus.

v) Inter-relationship amongst the line integral, double and triple integral formulations.

vi) Realize importance of Green, Gauss and Stokes' theorems in other branches of mathematics.

Unit-I: Sequences, Continuity and Differentiability

Notion of convergence of sequences and series of real numbers, ε-δ definition of limit and continuity of a real valued function; Differentiability and its geometrical interpretation; Rolle's theorem, Lagrange's mean value theorem, Cauchy's mean value theorem and their geometrical interpretations, Darboux's theorem.

Unit-II: Expansion of Functions

Successive differentiation and Leibnitz theorem, Maclaurin's and Taylor's theorems for expansion of a function, Taylor's theorem in finite form with Lagrange, Cauchy and Roche–Schlömilch forms of remainder.

Unit-III: Curvature, Asymptotes and Curve Tracing

Curvature; Asymptotes of general algebraic curves, Parallel asymptotes, Asymptotes parallel to axes; Symmetry, Concavity and convexity, Points of inflection, Tangents at origin, Multiple points, Position and nature of double points; Tracing of Cartesian, polar and parametric curves; Envelopes and evolutes.

Unit-IV: Functions of Several Variables

Limit, continuity and first order partial derivatives, Higher order partial derivatives, Change of variables, Euler's theorem for homogeneous functions, Taylor's theorem, Total differentiation and Jacobians.

Unit-V: Double and Triple Integrals

Double integration over rectangular and nonrectangular regions, Double integrals in polar co-ordinates, Triple integral over a parallelepiped and solid regions, Volume by triple integrals, Line integrals, Green's theorem, Area as a line integral, Surface integrals, Stokes' theorem, The Gauss divergence theorem.

References:

1. Howard Anton, I. Bivens & Stephan Davis (2016). *Calculus* (10th edition). Wiley India.
2. Gabriel Klambauer (1986). *Aspects of Calculus*. Springer-Verlag.
3. Wieslaw Krawcewicz & Bindhyachal Rai (2003). *Calculus with Maple Labs*. Narosa.
4. Gorakh Prasad (2016). *Differential Calculus* (19th edition). Pothishala Pvt. Ltd.
5. George B. Thomas Jr., Joel Hass, Christopher Heil & Maurice D. Weir (2018). *Thomas' Calculus* (14th edition). Pearson Education.
6. Jerrold Marsden, Anthony J. Tromba & Alan Weinstein (2009). *Basic Multivariable Calculus*, Springer India Pvt. Limited.
7. James Stewart (2012). *Multivariable Calculus* (7th edition). Brooks/Cole. Cengage.
8. Monty J. Strauss, Gerald L. Bradley & Karl J. Smith (2011). *Calculus* (3rd edition). Pearson Education. Dorling Kindersley (India) Pvt. Ltd.

Semester II

Paper-M201: Algebra

Course Learning Outcomes: This course will enable the students to:

i) Employ De Moivre's theorem in a number of applications to solve numerical problems.

ii) Learn about the fundamental concepts of groups, subgroups, normal subgroups, isomorphism theorems, cyclic and permutation groups.

iii) Recognize consistent and inconsistent systems of linear equations by the row echelon form of the augmented matrix, using rank.

iv) Find eigenvalues and corresponding eigenvectors for a square matrix.

v) Understand real vector spaces, subspaces, basis, dimension and their properties.

Unit-I: Set Theory and Theory of Equations

Sets, Relations, Equivalence relations, Equivalence classes; Finite, countable and uncountable sets; The division algorithm, Divisibility and the Euclidean algorithm, Modular arithmetic and basic properties of congruences; Elementary theorems on the roots of polynomial equations, Imaginary roots, The fundamental theorem of algebra (statement only); The n^{th} roots of unity, De Moivre's theorem for integer and rational indices and its applications.

Unit-II: Groups, Subgroups, Normal Subgroups and Isomorphism Theorems

Definition and properties of a group, Abelian groups, Examples of groups including D_n (dihedral groups), Q_8 (quarternian group), $GL(n,\mathbb{R})$ (general linear groups) and $SL(n,\mathbb{R})$ (special linear groups); Subgroups and examples, Cosets and their properties, Lagrange's theorem and its applications, Normal subgroups and their properties, Simple groups, Factors groups; Group homomorphisms and isomorphisms with properties; First, second and third isomorphism theorems for groups.

Unit-III: Cyclic and Permutation Groups

Cyclic groups and properties, Classifications of subgroup of cyclic groups, Cauchy theorem for finite Abelian groups; Centralizer, Normalizer, Center of a group, Product of two subgroups, Permutation group and properties, Even and odd permutations, Cayley's theorem.

Unit-IV: Row Echelon Form of Matrices and Applications

Systems of linear equations, Row reduction and echelon forms, The rank of a matrix and its applications in solving system of linear equations; Matrix operations, Symmetric, skew-

symmetric, self-adjoint, orthogonal, Hermition, skew-Hermition and unitary matrices; Determinant of a square matrix, The inverse of a square matrix, Eigenvectors and eigen values, The characteristic equation and the Cayley–Hamilton theorem, Applications of matrices to computer graphics and search engines.

Unit-V: Vector Spaces and Linear Transformations

Definitions of field and vector space with examples, Subspaces, Linear span, Quotient space and direct sum, Linearly independent and dependent sets, Bases and dimension, Linear transformation and matrix of a linear transformation, Change of coordinates, Rank and nullity of linear transformation, Rank-nullity theorem.

References:

1. Michael Artin (2014). *Algebra* (2nd edition). Pearson.
2. John B. Fraleigh (2007). *A First Course in Abstract Algebra* (7th edition). Pearson.
3. Stephen H. Friedberg, Arnold J.Insel& Lawrence E. Spence (2003). *Linear Algebra* (4thedition). Prentice-Hall of India Pvt. Ltd.
4. Joseph A. Gallian (2017). *Contemporary Abstract Algebra* (9th edition). Cengage.
5. Kenneth Hoffman & Ray Kunze (2015). *Linear Algebra* (2nd edition). Prentice-Hall.
6. I. N. Herstein (2006). *Topics in Algebra* (2nd edition). Wiley India.
7. Nathan Jacobson (2009). *Basic Algebra* I (2nd edition). Dover Publications.
8. Ramji Lal (2017). *Algebra 1: Groups, Rings, Fields and Arithmetic*. Springer.
9. I.S. Luthar & I.B.S. Passi (2013). *Algebra: Volume 1: Groups.* Narosa.

Semester-III

Paper-M301: Differential Equations

Course Learning Outcomes: The course will enable the students to:

i) Understand the genesis of ordinary as well as partial differential equations.

ii) Learn various techniques of getting exact solutions of certain solvable first order differential equations and linear differential equations of second order.

iii) Know Picard's method of obtaining successive approximations of solutions of first order ordinary differential equations, passing through a given point in the plane.

iv) Learn about solution of first order linear partial differential equations using Lagrange's method.

v) Know how to solve second order linear partial differential equations with constant coefficients.

vi) Formulate mathematical models in the form of ordinary and partial differential equations to problems arising in physical, chemical and biological disciplines.

Unit-I: First Order Differential Equations

Basic concepts and genesis of ordinary differential equations, Order and degree of a differential equation, Differential equations of first order and first degree, Equations in which variables are separable, Homogeneous equations, Linear differential equations and equations reducible to linear form, Exact differential equations, Integrating factor, First order higher degree equations solvable for x, y and p, Clairaut's form and singular solutions; Picard's method of successive approximations and the statement of Picard's theorem for the existence and uniqueness of the solutions of the first order differential equations.

Unit-II: Second Order Linear Differential Equations

Statement of existence and uniqueness theorem for the solution of linear differential equations, General theory of linear differential equations of second order with variable coefficients, Solutions of homogeneous linear ordinary differential equations of second order with constant coefficients, Method of variation of parameters and method of undetermined coefficients, Reduction of order, Euler-Cauchy equations, Coupled linear differential equations with constant coefficients.

Unit-III: First Order Partial Differential Equations

Genesis of Partial differential equations (PDE), Concept of linear and non-linear PDEs, Methods of solution of Simultaneous differential equations of the form: $dx/P(x,y,z) = dy/Q(x,y,z) = dz/R(x,y,z)$, Lagrange's method for PDEs of the form: $P(x,y,z)p+Q(x,y,z)q=R(x,y,z)$, where $p=\partial z/\partial x$ and $q=\partial z/\partial y$; Solutions passing through a given curve.

Unit-IV: Second Order Partial Differential Equations with Constant Coefficients

Principle of superposition for homogeneous linear PDEs, Relation between solution sets of non-homogeneous linear PDEs and their corresponding homogeneous equations, Reducible and irreducible homogeneous equations and their solutions in various possible cases, Solution of non-homogeneous reducible equations using Lagrange's method for first order equations.

Unit-V: Applications

Orthogonal trajectories of one-parameter families of curves in a plane, Minimum velocity of escape from Earth's gravitational field, Newton's law of cooling, Malthusian and logistic population models, Radioactive decay, Free and forced mechanical oscillations of a spring suspended vertically carrying a mass at its lowest tip, Phenomena of resonance, LCR circuits, Surfaces orthogonal to a given system of surfaces.

References:

1. Erwin Kreyszig (2011). *Advanced Engineering Mathematics* (10th edition). J. Wiley & Sons
2. B. Rai & D. P. Choudhury (2006). *Ordinary Differential Equations - An Introduction*. Narosa Publishing House Pvt. Ltd. New Delhi.
3. Shepley L. Ross (2007). *Differential Equations* (3rd edition). Wiley.
4. George F. Simmons (2017). *Differential Equations with Applications and Historical Notes* (3rd edition). CRC Press. Taylor & Francis.
5. Ian N. Sneddon (2006). *Elements of Partial Differential Equations*. Dover Publications.

Semester IV

Paper- M401 Real Analysis

Course Learning Outcomes: This course will enable the students to:

i) Understand basic properties of real number system such as least upper bound property and Order property.
ii) Realize importance of bounded, convergent, Cauchy and monotonic sequences of real numbers, find their limit superior and limit inferior.
iii) Apply various tests to determine convergence and absolute convergence of a series of real numbers.
iv) Learn about Riemann integrability of bounded functions and algebra of R-integrable functions.
v) Determine various applications of the fundamental theorem of integral calculus.
vi) Relate concepts of uniform continuity, differentiation, integration and uniform convergence.

Unit-I: Real Numbers

The set of real numbers ($\mathbb{R}$) as an ordered field, Least upper bound properties of $\mathbb{R}$**,** Metric property and completeness of $\mathbb{R}$, Archimedean property of $\mathbb{R}$, Dense subsets of $\mathbb{R}$, Nested intervals property; Neighborhood of a point in $\mathbb{R}$, Open sets, limit point of a set, closed and perfect sets in $\mathbb{R}$, connected and compact subsets of $\mathbb{R}$, Heine-Borel theorem.

Unit-II: Convergence of Sequences in $\mathbb{R}$

Bounded and monotonic sequences, Convergent sequence and its limit, Limit theorems, Monotone convergence theorem, Subsequences, Bolzano–Weierstrass theorem, Limit superior and limit inferior, Cauchy sequence, Cauchy's convergence criterion.

Unit-III: Infinite Series

Convergence of a series of positive real numbers, Necessary condition for convergence, Cauchy criterion for convergence; Tests for convergence: Comparison test, Limit comparison test, D'Alembert's ratio test, Cauchy's n^{th} root test, Abel's test, Integral test; Alternating series, Absolute and conditional convergence, Leibniz theorem, Rearrangements of series, Riemann's rearrangement theorem.

Unit-IV: Riemann Integration

Riemann integrability of bounded functions, Examples of R-integrable and non-integrable functions, Algebra of Riemann integrable functions, Integrability of continuous and monotonic functions, Darboux theorems, Fundamental theorem of integral calculus, First mean value theorem and second mean value theorems (Bonnet and Weierstrass forms). Necessary and sufficient condition for Riemann integrable function (Statement only).

Unit-V: Uniform Convergence, Continuity and Improper Integrals

Pointwise and uniform convergence of sequence and series of functions, Uniform continuity, Weierstrass's M-test, Uniform convergence and continuity, Uniform convergence and differentiability, Improper integrals and tests for improper integrals, Beta and Gamma functions.

References:

1. T. M. Apostol (2008). *Mathematical Analysis: A Modern Approach to Advanced Calculus*. Pearson Education.
2. Charalambos D. Aliprantis &) Owen Burkinshaw1998). *Principles of Real Analysis* (3rd edition). Academic Press.
3. Robert G. Bartle & Donald R. Sherbert (2015). *Introduction to Real Analysis* (4th edition).Wiley India.
4. Gerald G. Bilodeau, Paul R. Thie & G. E. Keough (2015). *An Introduction to Analysis* (2nd edition), Jones and Bartlett India Pvt. Ltd.
5. E. Hewitt & K. Stromberg (2013). *Real and Abstract Analysis.* Springer-Verlag.
6. K. A. Ross (2013). *Elementary Analysis*: *The Theory of Calculus* (2nd edition). Springer.
7. Walter Rudin. *Principles of Mathematical Analysis* (3rd edition), Tata McGraw Hill.

Semester V

Electives (Any one)

Paper-M501

Paper-(i): Mechanics

Course Learning Outcomes: This course will enable the students to:

i) Familiarize with subject matter, which has been the single centre, to which were drawn mathematicians, physicists, astronomers and engineers together.

ii) Understand necessary conditions for the equilibrium of particles acted upon by various forces and learn the principle of virtual work for a system of coplanar forces acting on a particle.

iii) Determine the centre of gravity of materialistic systems and discuss the equilibrium of a uniform cable hanging freely under its own weight.

iv) Deal with the kinematics and kinetics of the rectilinear and planar motions of a particle including the constrained oscillatory motions of particles.

v) Learn that a particle moving under a central force describes a plane curve and know the Kepler's laws of the planetary motions, which were deduced by him long before the mathematical theory given by Newton.

Unit-I: Statics

Coplanar forces, Couples, Moment of force and a couple about a point and a line, Equilibrium of a particle and of a system of particles; Work and potential energy, Principle of virtual work for a system of coplanar forces acting on a particle, Forces which can be omitted in forming the equations of virtual work.

Unit-II: Centre of Gravity and Common Catenary

Concepts of Centre of mass and Centre of gravity, Centre of gravity of an uniform arc, plane area and solids of revolution; Common catenary, Approximations of a catenary.

Unit-III: Rectilinear Motion

Simple harmonic motion and its geometrical representation, Motion under inverse square law, Motion in resisting media, Concept of terminal velocity, Motion of varying mass.

Unit-IV: Motion in a Plane

Kinematics and kinetics of motion, Expressions for velocity and acceleration in Cartesian, polar and intrinsic coordinates; Motion in a vertical circle, projectile and cycloidal motion.

Unit-V: Central Orbits

Equation of motion under a central force, Differential equation of an orbit, (p, r) equation of an orbit, Apses and apsidal distances, Areal velocity, Characteristics of central orbits, Kepler's laws of planetary motion.

References:

1. R. S. Varma (1962). *A Text Book of Statics*. Pothishala Pvt. Ltd.
2. P.L. Srivastava (1964). *Elementary Dynamics*. Ram Narain Lal, Beni Prasad Publishers Allahabad.
3. J. L. Synge & B. A. Griffith (1949). *Principles of Mechanics*. McGraw-Hill.
4. S.L. Loney (2006). *An Elementary Treatise on the Dynamics of a Particle and of Rigid Bodies*. Read Books.
5. A. S. Ramsey (2009). *Statics*. Cambridge University Press.
6. A. S. Ramsey (2009). *Dynamics*. Cambridge University Press.

Paper-(ii): Probability and Statistics

Course Learning Outcomes: This course will enable the students to:

i) Understand the basic concepts of probability.
ii) Appreciate the importance of probability distribution of random variables and to know the notion of central tendency.
iii) Establish the joint distribution of two random variables in terms their correlation and regression.
iv) Understand central limit theorem which shows that the empirical frequencies of so many natural populations exhibit normal distribution.
v) Study entropy and information theory in the framework of probabilistic models.

Unit-I: Probability and Random Variables

Axiomatic and empirical definitions of probability, Independent and dependent events, Conditional probability and Baye's theorem; Discrete and continuous random variables and their probability distributions, Cumulative distribution function, n^{th} Moments, Moment generating function, Characteristic function.

Unit-II: Univariate Distributions

Discrete distributions**:** Bernoulli trials and Bernoulli distribution, Binomial and Poisson distributions; Continuous distributions**:** Uniform, Geometric, Gamma, Exponential, Chi-square, Beta and normal distributions; Normal approximation to the binomial distribution, Central limit theorem.

Unit-III: Bivariate Distribution

Joint cumulative distribution function and its properties, Joint probability density function, Marginal distributions, Expectation of function of two random variables, Joint moment generating function, Conditional distributions and expectations, Independence of bivariate random variables.

Unit-IV: Correlation and Regression

The Correlation coefficient, Covariance, Calculation of covariance from joint moment generating function, Linear regression for two variables, The method of least squares, Bivariate normal distribution, Markov theorem, Chebyshev's theorem, Weak and strong laws of large numbers.

Unit-V: Information Theory

Uncertainty, Information and entropy, Conditional and joint entropy, Uniform Priors, Polya's urn model and random graphs, Applications of random graphs.

References:

1. David Applebaum (1996). *Probability and Information: An Integrated Approach.* Cambridge University Press.
2. Robert V. Hogg, Joseph W. McKean & Allen T. Craig (2013). *Introduction to Mathematical Statistics* (7th edition), Pearson Education.
3. Irwin Miller & Marylees Miller (2014). *John E. Freund's Mathematical Statistics with Applications* (8th edition). Pearson. Dorling Kindersley Pvt. Ltd. India.
4. Jim Pitman (1993). *Probability*, Springer-Verlag.
5. Sheldon M. Ross (2014). *Introduction to Probability Models* (11th edition). Elsevier.
6. A. M. Yaglom and I. M. Yaglom (1983). *Probability and Information*. D. Reidel Publishing Company. Distributed by Hindustan Publishing Corporation (India) Delhi.

Paper-(iii): Numerical Methods

Course Learning Outcomes: This course will enable the students to:

i) Obtain numerical solutions of algebraic and transcendental equations.

ii) Find numerical solutions of system of linear equations and to check the accuracy of the solutions.

iii) Learn about various interpolating and extrapolating methods to find numerical solutions.

iv) Solve initial and boundary value problems in differential equations using numerical methods.

v) Apply various numerical methods in real life problems.

Unit-I: Numerical Methods for Solving Algebraic and Transcendental Equations

Round-off error and computer arithmetic, Local and global truncation errors, Algorithms and convergence; Bisection method, false position method, fixed point iteration method, Newton's method and secant method for solving equations.

Unit-II: Numerical Methods for Solving Linear Systems

Partial and scaled partial pivoting, LU decomposition and its applications, Thomas method for tridiagonal systems; Gauss–Jacobi, Gauss–Seidel and successive over-relaxation (SOR) methods.

Unit-III: Interpolation

Lagrange and Newton interpolations, Piecewise linear interpolation, Cubic spline interpolation, Finite difference operators, Gregory–Newton forward and backward difference interpolations.

Unit-IV: Numerical Differentiation and Integration

First order and higher order approximation for first derivative, Approximation for second derivative; Numerical integration: Trapezoidal rule, Simpson's rule and its error analysis, Bulirsch–Stoer extrapolation methods, Richardson extrapolation.

Unit-V: Initial and Boundary Value Problems of Differential Equations

Euler’s method, Runge–Kuttamethods, Higher order one step method, Multi-step methods; Finite difference method, Shooting method, Real life examples: Google search engine, 1D and 2D simulations, Weather forecasting.

References:

1. Brian Bradie (2006), *A Friendly Introduction to Numerical Analysis.* Pearson.
2. C. F. Gerald & P. O. Wheatley (2008). *Applied Numerical Analysis* (7th edition), Pearson Education, India.
3. M.K. Jain, S. R. K. Iyengar & R. K. Jain (2012). *Numerical Methods for Scientific and Engineering Computation* (6th edition). New Age International Publishers.
4. Robert J. Schilling & Sandra L. Harris (1999). *Applied Numerical Methods for Engineers Using* MATLAB *and C*. Thomson-Brooks/Cole.

Paper- (iv): Complex Variables

Course Learning Outcomes: This course will enable the students to:

i) Visualize complex numbers as points of $\mathbb{R}^2$, stereographic projection of complex plane on the Riemann sphere and various geometric properties of linear fractional transformations.

ii) Understand the significance of differentiability and analyticity of complex functions leading to the Cauchy–Riemann equations.

iii) Learn the role of Cauchy–Goursat theorem and Cauchy integral formula in evaluation of contour integrals.

iv) Apply Liouville's theorem in fundamental theorem of algebra.

v) Understand the convergence, term by term integration and differentiation of a power series.

vi) Learn Taylor and Laurent series expansions of analytic functions; classify the nature of singularities, poles and residues and application of Cauchy Residue theorem.

Unit-I: Complex Plane

Complex numbers and their representation, algebra of complex numbers; Complex plane, Open set, Domain and region in complex plane; Stereographic projection and Riemann sphere, Complex functions and their limits including limit at infinity; Continuity, Möbius transformations and their geometrical properties.

Unit-II: Analytic Functions and Cauchy–Riemann Equations

Complex functions and their limits including limit at infinity; Continuity, differentiability and analyticity; Cauchy–Riemann equations, Harmonic functions, Sufficient conditions for differentiability and analyticity, Analyticity and zeros of exponential, trigonometric and logarithmic functions; Branch cut and branch of multi-valued functions.

Unit-III: Cauchy's Theorems and Fundamental Theorem of Algebra

Line integral, Path independence, Complex integration, Green's theorem, Anti-derivative theorem, Cauchy–Goursat theorem, Cauchy integral formula, Cauchy's inequality, Derivative of analytic function, Liouville's theorem, Fundamental theorem of algebra, Maximum modulus theorem and its consequences.

Unit-IV: Power Series

Sequences, series and their convergence, Taylor series and Laurent series of analytic functions, Power series, Radius of convergence, Integration and differentiation of power series, Absolute and uniform convergence of power series.

Unit-V: Singularities and Contour Integration

Zeros and poles of meromorphic functions, Nature of singularities, Picard's theorem, Residues, Cauchy's residue theorem, Argument principle, Rouche's theorem, Jordan's lemma, Evaluation of proper and improper integrals.

References:

1. Lars V. Ahlfors (2017). *Complex Analysis* (3rd edition). McGraw-Hill Education.
2. Joseph Bak & Donald J. Newman (2010). *Complex Analysis* (3rd edition). Springer.
3. James Ward Brown & Ruel V. Churchill (2009). *Complex Variables and Applications* (9th edition). McGraw-Hill Education.
4. John B. Conway (1973). *Functions of One Complex Variable*. Springer-Verlag.
5. E.T. Copson (1970). *Introduction to Theory of Functions of Complex Variable*. Oxford University Press.
6. Theodore W. Gamelin (2001). *Complex Analysis*. Springer-Verlag.
7. George Polya & Gordon Latta (1974). Complex Variables. Wiley.
8. H. A. Priestley (2003). Introduction to Complex Analysis. Oxford University Press.
9. E. C. Titchmarsh (1976). Theory of Functions (2nd edition). Oxford University Press.

Paper-(v): Linear Algebra

Course Learning Outcomes: This course will enable the students to:

i) Learn about properties of linear transformation and isomorphism theorems.
ii) Understand the concept of polynomials and their prime factorization.
iii) Find canonical form of linear transformations.
iv) Obtain various variants of diagonalisation of linear transformations.
v) Apply Cauchy-Schwarz inequality for deriving metric on inner product spaces and obtain orthonormal basis using Gram-Schmidt orthogonalisation.

Unit-I: Properties of Linear Transformation

Vector spaces, Linearly independent and dependent sets, Bases and dimension, Linear transformation, Linear functional, Dual spaces and second dual space, Transpose of linear transformation, Algebra of linear transformations, Isomorphism theorems.

Unit-II: Polynomials

Algebras, The algebra of polynomials, Lagrange interpolation, Vandermonde matrix, Polynomial ideals, Taylor's formula, The prime factorization of a polynomial, Algebraically closed fields.

Unit-III: Elementary Cannonical Forms

Determinant functions, Characteristic values of a linear transformation, Cayley-Hamilton theorem for linear transformations, Annihilating polynomials, Invariant subspaces, Minimal and characteristic polynomials.

Unit-IV: Diagonalisation and Jordan Cannonical Form

Diagonalisability of linear transformations, Direct sum decomposition, Invariant direct sums, The primary decomposition theorem, Triangular form, Jordan canonical form, trace and transpose.

Unit-V: Inner Product Spaces

Definition and examples of inner product space, orthogonality, Cauchy-Schwarz inequality, Gram-Schmidt orthogonalisation, Diagonalisation of symmetric matrices, Hermitian, Unitary and normal operators.

References:

1. Stephen H.Friedberg, Arnold J.Insel& Lawrence E. Spence (2003). *Linear Algebra* (4thedition). Prentice-Hall of India Pvt. Ltd.

2. I. M. Gel'fand (1989). *Lectures on Linear Algebra*. Dover Publications.
3. Kenneth Hoffman & Ray Kunze (2015). *Linear Algebra* (2nd edition). Prentice-Hall.
4. Nathan Jacobson (2009). *Basic Algebra* I (2nd edition). Dover Publications.
5. Nathan Jacobson (2009). *Basic Algebra* II (2nd edition). Dover Publications.
6. Serge Lang (2005). *Introduction to Linear Algebra* (2nd edition). Springer India.
7. Gilbert Strang (2014). *Linear Algebra and its Applications* (2ndedition). Elsevier.

Paper-(vi): Integral Transforms and Fourier Analysis

Course Learning Outcomes: This course will enable the students to:

i) Know about piecewise continuous functions, Dirac delta function, Laplace transforms and its properties.

ii) Solve ordinary differential equations using Laplace transforms.

iii) Familiarise with Fourier transforms of functions belonging to $L^1(\mathbb{R})$ class, relation between Laplace and Fourier transforms.

iv) Explain Parseval's identity, Plancherel's theorem and applications of Fourier transforms to boundary value problems.

v) Learn Fourier series, Bessel's inequality, term by term differentiation and integration of Fourier series.

Unit-I: Laplace Transforms

Integral transform, Kernel of an integral transform, Reduction of integral transform into Laplace transform, Linearity, Existence theorem, Laplace transforms of derivatives and integrals, Shifting theorems, Change of scale property, Laplace transforms of periodic functions, Dirac's delta function.

Unit-II: Further Properties of Laplace Transforms and Applications

Differentiation and integration of transforms, Convolution theorem, Integral equations, Inverse Laplace transform, Lerch's theorem, Linearity property of inverse Laplace transform, Translations theorems of inverse Laplace transform, Inverse transform of derivatives, Applications of Laplace transform in obtaining solutions of ordinary differential equations and integral equations.

Unit-III: Fourier Transforms

Fourier and inverse Fourier transforms, Fourier sine and cosine transforms, Inverse Fourier sine and cosine transforms, Linearity property, Change of scale property, Shifting property, Modulation theorem, Relation between Fourier and Laplace transforms.

Unit-IV: Solution of Equations by Fourier Transforms

Solution of integral equation by Fourier sine and cosine transforms, Convolution theorem for Fourier transform, Parseval's identity for Fourier transform, Plancherel's theorem, Fourier transform of derivatives, Applications of infinite Fourier transforms to boundary value problems, Finite Fourier transform, Inversion formula for finite Fourier transforms.

Unit-V: Fourier Series

Fourier cosine and sine series, Fourier series, Differentiation and integration of Fourier series, Absolute and uniform convergence of Fourier series, Bessel's inequality, The complex form of Fourier series.

References:

1. James Ward Brown & Ruel V. Churchill (2011). F*ourier Series and Boundary Value Problems*. McGraw-Hill Education.
2. Charles K. Chui (1992). *An Introduction to Wavelets*. Academic Press.
3. Erwin Kreyszig (2011). *Advanced Engineering Mathematics* (10^{th} edition). Wiley.
4. Walter Rudin (2017). *Fourier Analysis on Groups*. Dover Publications.
5. A. Zygmund (2002). *Trigonometric Series* (3^{rd} edition). Cambridge University Press.

Semester VI

Electives (Any one)

Paper – M601

Paper-(i): Discrete Mathematics

Course Learning Outcomes: This course will enable the students to:

i) Learn about partially ordered sets, lattices and their types.

ii) Understand Boolean algebra and Boolean functions, logic gates, switching circuits and their applications.

iii) Solve real-life problems using finite-state and Turing machines.

iv) Assimilate various graph theoretic concepts and familiarize with their applications.

Unit-I: Partially Ordered Sets

Definitions, examples and basic properties of partially ordered sets (poset), Order isomorphism, Hasse diagrams, Dual of a poset, Duality principle, Maximal and minimal elements, Least upper bound and greatest upper bound, Building new poset, Maps between posets.

Unit-II: Lattices

Lattices asposets, Lattices as algebraic structures, Sublattices, Products and homomorphisms; Definitions, examples and properties of modular and distributive lattices; Complemented, relatively complemented and sectionally complemented lattices.

Unit-III: Boolean Algebras and Switching Circuits

Boolean algebras, De Morgan's laws, Boolean homomorphism, Representation theorem; Boolean polynomials, Boolean polynomial functions, Disjunctive and conjunctive normal forms, Minimal forms of Boolean polynomials, Quine–McCluskeymethod, Karnaugh diagrams, Switching circuits and applications.

Unit-IV: Finite-State and Turing Machines

Finite-state machines with outputs, and with no output; Deterministic and nodeterministic finite-state automaton; Turing machines: Definition, examples, and computations.

Unit-V: Graphs

Definition, examples and basic properties of graphs, Königsberg bridge problem; Subgraphs, Pseudographs, Complete graphs, Bipartite graphs, Isomorphism of graphs, Paths and circuits,

Eulerian circuits, Hamiltonian cycles, Adjacency matrix, Weighted graph, Travelling-salesman problem, Shortest path, Dijkstra's algorithm.

References:

1. B. A. Davey & H. A. Priestley (2002). *Introduction to Lattices and Order* (2nd edition). Cambridge University Press.
2. Edgar G. Goodaire& Michael M. Parmenter (2018). *Discrete Mathematics with Graph Theory* (3rd edition). Pearson Education.
3. Rudolf Lidl & Günter Pilz (1998). *Applied Abstract Algebra* (2nd edition). Springer.
4. Kenneth H. Rosen (2012). *Discrete Mathematics and its Applications*: *With Combinatorics and Graph Theory* (7th edition). McGraw-Hill.
5. C. L. Liu (1985). *Elements of Discrete Mathematics* (2nd edition). McGraw-Hill.

Paper-(ii): Linear Programming and Game Theory

Course Learning Outcomes: This course will enable the students to:

i) Analyze and solve linear programming models of real life situations.
ii) Provide graphical solution of linear programming problems with two variables, and illustrate the concept of convex set and extreme points.
iii) Solve linear programming problems using simplex method.
iv) Learn techniques to solve transportation and assignment problems.
v) Solve two-person zero sum game problems.

Unit-I: Linear Programming Problem, Convexity and Basic Feasible Solutions

Formulation and examples**,** Canonical and Standard forms, Graphical solution, Convex and polyhedral sets, Extreme points, Basic solutions, Basic Feasible Solutions, Correspondence between basic feasible solutions and extreme points.

Unit-II: Simplex Method

Optimality criterion, Improving a basic feasible solution, Unboundedness; Simplex algorithm and its tableau format; Artificial variables, Two-phase method, Big-*M* method.

Unit-III: Duality

Formulation of the dual problem, Duality theorems, Unbounded and infeasible solutions in the primal, Solving the primal problem using duality theory.

Unit-IV: Transportation and Assignment Problems

Formulation of transportation problems, Methods of finding initial basic feasible solutions: North-west corner rule, Least cost method, Vogel approximation method, Algorithm for obtaining optimal solution; Formulation of assignment problems, Hungarian method.

Unit-V: Game Theory

Formulation of two-person zero-sum games, Games with mixed strategies, Graphical method for solving matrix game, Dominance principle, Solution of game problem, Linear programming method of solving a game.

References:

1. Mokhtar S. Bazaraa, John J. Jarvis & Hanif D. Sherali (2010). *Linear Programming and Network Flows* (4th edition). John Wiley & Sons.
2. G. Hadley (2002). *Linear Programming*. Narosa Publishing House.

3. Frederick S. Hillier & Gerald J. Lieberman (2015). *Introduction to Operations Research* (10th edition). McGraw-Hill Education.
4. Hamdy A. Taha (2017). *Operations Research*: *An Introduction* (10th edition). Pearson.
5. Paul R. Thie & Gerard E. Keough (2014). *An Introduction to Linear Programming and Game Theory* (3rd edition). Wiley India Pvt. Ltd.

Paper-(iii): Tensors and Differential Geometry

Course Learning Outcomes: This course will enable the students to:

i) Explain the basic concepts of tensors.
ii) Understand role of tensors in differential geometry.
iii) Learn various properties of curves including Frenet–Serret formulae and their applications.
iv) Know the Interpretation of the curvature tensor, Geodesic curvature, Gauss and Weingarten formulae.
v) Understand the role of Gauss's Theorema Egregium and its consequences.
vi) Apply problem-solving with differential geometry to diverse situations in physics, engineering and in other mathematical contexts.

Unit-I: Tensors

Contravariant and covariant vectors, Transformation formulae, Tensor product of two vector spaces, Tensor of type (r, s), Symmetric and skew-symmetric properties, Contraction of tensors, Quotient law, Inner product of vectors.

Unit-II: Further Properties of Tensors

Fundamental tensors, Associated covariant and contravariant vectors, Inclination of two vectors and orthogonal vectors, Christoffel symbols, Law of transformation of Christoffel symbols, Covariant derivatives of covariant and contravariant vectors, Covariant differentiation of tensors, Curvature tensor, Ricci tensor, Curvature tensor identities.

Unit-III: Curves in $\mathbb{R}^2$ and $\mathbb{R}^3$

Basic definitions and examples, Arc length, Curvature and the Frenet–Serret formulae, Fundamental existence and uniqueness theorem for curves, Non-unit speed curves.

Unit-IV: Surfaces in $\mathbb{R}^3$

Basic definitions and examples, The first fundamental form, Arc length of curves on surfaces, Normal curvature, Geodesic curvature, Gauss and Weingarten formulae, Geodesics, Parallel vector fields along a curve and parallelism.

Unit-V: Geometry of Surfaces

The second fundamental form and the Weingarten map; Principal, Gauss and mean curvatures; Isometries of surfaces, Gauss's Theorema Egregium, The fundamental theorem of surfaces, Surfaces of constant Gauss curvature, Exponential map, Gauss lemma, Geodesic coordinates, The Gauss–Bonnet formula and theorem.

References:

1. Christian Bär (2010). *Elementary Differential Geometry*. Cambridge University Press.
2. Manfredo P. do Carmo (2016). *Differential Geometry of Curves & Surfaces* (Revised and updated 2nd edition). Dover Publications.
3. Alferd Gray (2018). *Modern Differential Geometry of Curves and Surfaces with Mathematica* (4th edition). Chapman & Hall/CRC Press, Taylor & Francis.
4. Richard S. Millman & George D. Parkar (1977). *Elements of Differential Geometry*. Prentice-Hall.
5. R. S. Mishra (1965). *A Course in Tensors with Applications to Riemannian Geometry*. Pothishala Pvt. Ltd.
6. Sebastián Montiel & Antonio Ross (2009). *Curves and Surfaces*. American Mathematical Society.

Paper-(iv): Number Theory

Course Learning Outcomes: This course will enable the students to learn:

i) Some of the open problems related to prime numbers, viz., Goldbach conjecture etc.
ii) About number theoretic functions and modular arithmetic.
iii) Public crypto systems, in particular, RSA.

Unit-I: Distribution of Primes and Theory of Congruencies

Linear Diophantine equation, Prime counting function, Prime number theorem, Goldbach conjecture, Fermat and Mersenne primes, Congruence relation and its properties, Linear congruence and Chinese remainder theorem, Fermat's little theorem, Wilson's theorem.

Unit-II: Number Theoretic Functions

Number theoretic functions for sum and number of divisors, Multiplicative function, The Mobius inversion formula, The greatest integer function. Euler’s phi-function and properties, Euler’s theorem.

Unit-III: Primitive Roots

The order of an integer modulo n, Primitive roots for primes, Composite numbers having primitive roots; Definition of quadratic residue of an odd prime, and Euler’s criterion.

Unit-IV: Quadratic Reciprocity Law and Public Key Encryption

The Legendre symbol and its properties, Quadratic reciprocity, Quadratic congruencies with composite moduli.

Unit-V: Applications

Public key encryption, RSA encryption and decryption, Some important application.

References:

1. David M. Burton (2007). *Elementary Number Theory* (7th edition). McGraw-Hill.
2. Gareth A. Jones & J. Mary Jones (2005). *Elementary Number Theo*ry. Springer.
3. Neville Robbins (2007). *Beginning Number Theory* (2nd edition). Narosa.

Paper-(v): Advanced Mechanics

Course Learning Outcomes: This course will enable the students to:

i) Understand the reduction of force system in three dimensions to a resultant force acting at a base point and a resultant couple.

ii) Learn about a nul point, a nul line, and a nul plane with respect to a system of forces acting on a rigid body together with the idea of central axis.

iii) Know the inertia constants for a rigid body and the equation of momental ellipsoid together with the idea of principal axes and principal moments of inertia to derive Euler's dynamical equations.

iv) Study the kinematics and kinetics of fluid motions to understand the equation of continuity in Cartesian, cylindrical polar and spherical polar coordinates which are used to derive Euler's equations and Bernoulli's equation.

v) Deal with two-dimensional fluid motion using the complex potential and also to understand the concepts of sources, sinks, doublets and the image systems of these with regard to a line and a circle.

Unit-I: Statics in Space

Forces in three dimensions, Reduction to a force and a couple, Equilibrium of a system of particles, Central axis and Wrench, Equation of the central axis, Nul points, nul lines and nul planes with respect to a given system of forces.

Unit-II: Motion of a Rigid Body

Definition of rigid body as a system of particles and condition of rigidity, Moments and products of inertia of standard bodies, Momental ellipsoid, Principal axes and principal moments of inertia; The momentum of a rigid body in terms of linear momentum and angular momentum about any point, Equations of motion in terms of linear and angular momenta, Motion of a rigid body with a fixed point, Existence of an angular velocity, Kinetic energy and angular momentum of a rigid body in terms of inertia constants, Euler's dynamical equations and the motion under no forces.

Unit-III: Kinematics of Fluid Motion

Lagrangian and Eulerian approaches, Acceleration of fluid at a point, Equation of continuity in Cartesian, cylindrical polar and spherical polar coordinates, Boundary surface, Streamlines

and path lines, Velocity potential, Rotational and irrotational motion, Vorticity vector and vortex lines.

Unit-IV: Kinetics of Fluid Motion

Euler's equations of motion in Cartesian, cylindrical polar and spherical polar coordinates, Bernoulli's equation, Impulsive motion.

Unit-V: Motion in Two-Dimensions

Stream function, Complex potential, Basic singularities: Sources, sinks, doublets and complex potentials due to these basic singularities; Image system of a simple source and a simple doublet with regard to a line and a circle.

References:

1. A. S. Ramsay (1960). *A Treatise on Hydromechanics, Part-II Hydrodynamics* G. Bell & Sons.
2. F. Chorlton (1967). *A Textbook of Fluid Dynamics*. CBS Publishers.
3. Michel Rieutord (2015). *Fluid Dynamics An Introduction.* Springer.
4. E. A. Milne (1965). *Vectorial Mechanics*,Methuen & Co.Limited.London.
5. F. Chorlton (1969). *A Text Book of Dynamics*, D Van Nosterand Co. Ltd.London.

Paper-(vi): Information Theory and Coding

Course Learning Outcomes: This course will enable the students to:

i) Study simple ideal statistical communication models.
ii) Understand the development of codes for transmission and detection of information.
iii) Learn about the input and output of a signal via transmission channel.
iv) Study detection and correction of errors during transmission.
v) Represent a linear code by matrices - encoding and decoding.

Unit-I: Concepts of Information Theory

Communication processes, A model of communication system, A quantitative measure of information, Binary unit of information, A measure of uncertainty, H function as a measure of uncertainty, Sources and binary sources, Measure of information for two-dimensional discrete finite probability schemes.

Unit-II: Entropy Function

A sketch of communication network, Entropy, Basic relationship among different entropies, A measure of mutual information, Interpretation of Shannon's fundamental inequalities; Redundancy, efficiency, and channel capacity; Binary symmetric channel, Binary erasure channel, Uniqueness of the entropy function, Joint entropy and conditional entropy, Relative entropy and mutual information, Chain rules for entropy, Conditional relative entropy and conditional mutual information, Jensen's inequality and its characterizations, The log sum inequality and its applications.

Unit-III: Concepts of Coding

Block codes, Hamming distance, Maximum likelihood decoding, Levels of error handling, Error correction, Error detection, Erasure correction, Construction of finite fields, Linear codes, Matrix representation of linear codes, Hamming codes.

Unit-IV: Bounds of Codes

Orthogonality relation, Encoding and decoding of linear codes, The singleton bound and maximum distance separable codes, The sphere-packing bound and perfect codes, The Gilbert–Varshamov bound, MacWilliams' identities.

Unit-V: Cyclic Codes

Definition and examples of cyclic codes, Generator polynomial and check polynomial, Generator matrix and check matrix, Bose–Chaudhuri–Hocquenghem (BCH) code as a cyclic code.

References:

1. Robert B. Ash, (2014). *Information Theory*. Dover Publications.
2. Thomas M. Cover & Joy A. Thomas (2013). *Elements of Information Theory* (2nd edition). Wiley India Pvt. Ltd.
3. Joseph A. Gallian (2017). *Contemporary Abstract Algebra* (9th edition), Cengage.
4. Fazlollah M. Reza, (2003). *An Introduction to Information Theory*. Dover Publications.
5. Ron M. Roth (2007). *Introduction to Coding Theory*. Cambridge University Press.
6. Claude E. Shannon & Warren Weaver (1969). *The Mathematical Theory of Communication*. The University of Illinios Press.

Paper-(vii): Special Theory of Relativity

Course Learning Outcomes: This course will enable the students to:

i) Understand the basic elements of Newtonian mechanics including Michelson–Morley experiment and geometrical interpretations of Lorentz transformation equations.
ii) Learn about length contraction, time dilation and Lorentz contraction factor.
iii) Study 4-dimensional Minkowskian space-time and its consequences.
iv) Understand equations of motion as a part of relativistic mechanics.
v) Imbibe connections between relativistic mechanics and electromagnetism.

Unit-I: Newtonian Mechanics

Inertial frames, Speed of light and Gallilean relativity, Michelson–Morley experiment, Lorentz–Fitzgerold contraction hypothesis, Relative character of space and time, Postulates of special theory of relativity, Lorentz transformation equations and its geometrical interpretation, Group properties of Lorentz transformations.

Unit-II: Relativistic Kinematics

Composition of parallel velocities, Length contraction, Time dilation, Transformation equations for components of velocity and acceleration of a particle and Lorentz contraction factor.

Unit-III: Geometrical representation of space-time

Four dimensional Minkowskian space-time of special relativity, Time-like, light-like and space-like intervals, Null cone, Proper time, World line of a particle, Four vectors and tensors in Minkowiskian space-time.

Unit-IV: Relativistic Mechanics

Variation of mass with velocity. Equivalence of mass and energy. Transformation equations for mass momentum and energy. Energy-momentum four vector. Relativistic force and Transformation equations for its components. Relativistic equations of motion of a particle.

Unit-V: Electromagnetism

Transformation equations for the densities of electric charge and current. Transformation equations for electric and magnetic field strengths. The Field of a Uniformly Moving Point charge. Forces and fields near a current carrying wire. Forces between moving charges. The invariance of Maxwell`s equations.

References:

1. James L. Anderson (1973). *Principles of Relativity Physics*. Academic Press.
2. Peter Gabriel Bergmann (1976). *Introduction to the Theory of Relativity*. Dover Publications.
3. C. Moller (1972). *The Theory of Relativity* (2nd edition). Oxford University Press.
4. Robert Resnick (2007). *Introduction to Special Relativity*. Wiley.
5. Wolfgang Rindler (1977). *Essential Relativity: Special, General, and Cosmological*. Springer-Verlag.
6. V. A. Ugarov (1979). *Special Theory of Relativity*. Mir Publishers, Moscow.

Paper-(viii): C++ Programming for Mathematics

Course Learning Outcomes: This course will enable the students to:

i) Understand and apply the programming concepts of C++ for solving mathematical problems.

ii) Apply to find greatest common divisors, generate random numbers, understand Cartesian geometry and algebraic concepts through programming.

iii) Represent the outputs of programs visually in terms of well formatted text and plots.

Course Contents:

Unit 1: Essentials of C++

Basics of programming, C++ as a general purpose programming language, Structure of a C++ program, Common compilers and IDE's, Basic data-types, Variables and literals in C++, Operators, Expressions, Evaluation precedence, Type compatibility, Debugging and testing; Finding greatest common divisor, Random number generation.

Unit 2: Structured Data

Structured data-types in C++, Arrays and manipulating data in arrays, Factorization of an integer, Compute Euler's totient; Objects and classes**:** Information hiding, Modularity, Constructors and destructors, Methods, Polymorphism; Cartesian geometry using points (2 & 3-dimensional), Pythagorean triples.

Unit 3: Containers and Templates

Containers and Template Libraries**:** Sets, Iterators, Multisets, Vectors, Maps, Lists, Stacks, Queues; Basic set algebra, Modulo arithmetic, Permutations, and Polynomials.

Unit 4: Mathematical Libraries and Packages

Arbitrary precision arithmetic using the GMP package; Two-dimensional arrays in C++ with applications in finding eigenvalues, eigenvectors, rank, nullity, and solving system of linear equations in matrices; Features of C++ for input/output and visualization**,** Strings, Streams, Formatting methods, Processing files in a batch, Command-line arguments, Visualization packages and their use in plots.

Unit-V: Odds and Ends

Runtime errors and graceful degradation, Robustness in a program; Exception handling: Try-catch and throw; Defining and deploying suitable exception handlers in programs; Compiler options; Conditional compilation; Understanding and defining suitable pragmas;

Identification and description of install parameters of mathematical libraries, debugging installation, working with multiple libraries simultaneously and maintaining correctness and consistency of data.

References:

1. Nell Dale & Chip Weems (2013).*Programming and Problem Solving with C++* (6th edition). Jones & Bartlett Learning.
2. Peter Gottschling (2016). *Discovering Modern C++*: *An Intensive Course for Scientists, Engineers, and Programmers*. Pearson.
3. Nicolai M. Josuttis (2012). *The C++ Standard Library*: *A Tutorial and Reference* (2nd edition). Addison-Wesley, Pearson.
4. Donald E. Knuth (1968). *The Art of Computer Programming.* Addison-Wesley.
5. Edward Scheinerman (2006). *C++ for Mathematicians*: *An Introduction for Students and Professionals*. Chapman & Hall/CRC. Taylor & Francis.
6. B. Stroustrup (2013). *The C++ Programming Language* (4th edition). Addison-Wesley.

6.3. References for each course

References for each course are given at the end of course contents of each course.

7. Teaching-Learning Process

The teaching-learning process should be aimed at systematic exposition of basic concepts so as to acquire knowledge of mathematics in a canonical manner. In this context, applications of mathematics and linkage with the theory constitute a vital aspect of the teaching-learning process. The course offers many modes of learning and assessment. Students have great freedom of choice of subjects which they can study. The various components of teaching-learning process are summarized in the following heads.

1. **Lectures:** The most common method of imparting knowledge is through lectures. There are diverse modes of delivering lectures such as through blackboard, power point presentation and other technology aided means. A judicious mix of these means is a key aspect of teaching-learning process.

2. **Tutorials:** Assimilating mathematical ideas, deepening understanding, and gaining mastery of new concepts all take time, commitment, and intelligent effort. To reinforce learning, to monitor progress, and to provide a regular pattern of study, tutorials are essential requirements. During these tutorials, difficulties faced by the students in understanding the lectures, are dealt with. Tutorials are also aimed at solving problems associated with the concepts discussed during the lectures.

3. **Practicals:** To give a geometrical visualisation and obtaining numerical solutions of mathematical problems, various Computer Algebra Systems (CAS) are used in practical sessions. These sessions provide vital insights into mathematical concepts and draw learner’s attention towards limitations of numerical computations. During practicals, mathematical models arising in real life problems can also be simulated.

4. **Options System:** LOCF in mathematics provides great flexibility both in terms of variety of courses and range of references in each course. In fifth and sixth semesters students can opt for elective courses from a wide range of pure and applied courses, depending on their interests and requirements.

5. **Field based learning:** Students may enhance their knowledge through field based learning while understanding the practical importance of mathematics especially in industries.

6. **Prescribed textbooks:** A large number of books are included in the list of references of each course for enrichment and enhancement of knowledge.

7. **E-learning resources:** Learner may also access electronic resources and educational websites for better understanding and updating the concepts.

8. **Self-study materials:** Self-study material provided by the teachers/instructors is an integral part of learning mathematics. It helps in bridging the gaps in the classroom teaching. It also provides scope for teachers to give additional information beyond classroom learning.

9. **Open-ended projects:** Home assignments at regular intervals and project work involving applications of theory are necessary to assimilate basic concepts of mathematics. Hence, it is incumbent on the part of a learner to complete open-ended projects assigned by the teacher.

10. **Internships:** The teaching-learning process needs to be further supported by other activities devoted to subject-specific and interdisciplinary skills, summer and winter internships in mathematics. During these internships it is expected that a learner will interact with experts and write a report on a topic provided to the learner.

11. **Institute visits:** Institute visit by a learner is also a part of learning process. During such visits a learner has access to knowledge by attending academic activities such as seminars, colloquia, library consultation and discussion with faculty members. These activities provide guidance and direction for further study.

12. **Industrial visits:** Industrial visits offer an opportunity to observe real time applications of mathematical concepts. These visits also give an opportunity to realise the power of mathematical ideas and their translation in problem solving.

13. **Training programmes:** Training programmes such as Mathematics Training and Talent Search (MTTS) program, organised by various agencies/institutes like National Board for Higher Mathematics, also provide an opportunity to learn various dimensions of mathematics.

8. Assessment Methods

A range of assessment methods which are appropriate to test the understanding of various concepts of mathematics will be used. Priority will be given to formative assessment. Various learning outcomes will be assessed using time-bound examinations, series of open and closed book tests with uniform distribution over time, problem solving, home assignments, individual and group project reports, seminar presentations, viva-voce examination, participation in mathematical quizzes/competitions at local, regional, national and international levels and participations in internship programs. For various courses in mathematics, the following assessment methods shall be adopted:

i. Announced/unannounced quizzes
ii. Scheduled/unscheduled tests
iii. Problem solving sessions aligned with classroom lectures
iv. Practical assignments
v. Regular chamber consultation with faculty members
vi. Periodic tests, mid semester examination and semester end comprehensive examination
vii. Seminar presentations
viii. Computer skill test and computer simulation of concepts learnt
ix. Awareness tests of historical development of mathematical ideas
x. Awareness tests of recent advances in mathematics
xi. Awareness tests of various national/international prizes in mathematics including Fields Medal, Abel prize, Rolf Nevanlinna Prize, Srinivasa Ramanujan Medal etc. and the work of recipients of these prizes
xii. Awareness test of applications of mathematics in other branches of science, technology and other disciplines.

9. Keywords

LOCF, CBCS, Course Learning Outcomes, Employability, Simulation, Graduate Attributes Communication Skills, Critical Thinking, Descriptors.