Proposed Syllabus and Scheme of Examination

for

B.Sc. (Hons.) Applied Mathematics

submitted to

University Grants Commission
New Delhi

under the

Choice Based Credit System

May 2015

Proposed Scheme for Choice Based Credit System in

B.Sc. (Hons.)Applied Mathematics

Semester	Core Course (14)	Ability Enhancement Compulsory Course (AECC)(2)	Skill Enhancement Course (SEC) (2)	Discipline Specific Elective (DSE) (4)	Generic Elective (GE) (4)
1	C1 Calculus (P) C2 Algebra	AECC1			GE1
2	C3 Real Analysis C4 Differential Equations (P)	AECC2			GE2
3	C5 Theory of Real Functions C6 Group Theory C7 PDE and Systems of ODE (P)		SEC1		GE3
4	C8 Multivariate Calculus C9 Complex Analysis C10 Rings and Linear Algebra		SEC2		GE4
5	C11 Mechanics C12 Numerical Methods and Programming (P)			DSE-1 DSE-2	

6	**C13 Integral Equations and Calculus of Variation**			**DSE-3**	
	C14 Laplace Transform (P)			**DSE-4**	

(P) *means course with practicals*

Note: Institutions opting for both B.Sc. (Hons.) Mathematics and B.Sc. (Hons.) Applied Mathematics shall have first two semesters common.

Discipline Specific Electives (DSE)

Choices for DSE 1 (choose one)

1. Number Theory
2. Graph Theory
3. Linear Programming

Choices for DSE 2 (choose one)

1. Control Theory
2. Approximation Theory
3. Combinatorial Optimization

Choices for DSE 3 (choose one)

1. Mathematical Modeling
2. Coding Theory
3. Wavelet Theory

Choices for DSE 4 (choose one)

1. Bio-Mathematics
2. Stochastic Processes
3. Difference Equations

Skill Enhancement Course (SEC)

Choices for SEC 1 (choose one)

1. Electronic Commerce
2. Computer Graphics (P)
3. Operating Systems

Choices for SEC 2 (choose one)

1. Latex and Web Designing (P)
2. Transportation and Game Theory
3. Fuzzy Sets and Logics

Generic Electives (GE)

Choices for GE 1 (choose one)

1. Object Oriented Programming in C++
2. Finite Element Methods

Choices for GE 2 (choose one)

1. Mathematical Finance
2. Econometrics

Choices for GE 3 (choose one)

1. Digital Signal Processing
2. Neural Networks
3. Dynamical Systems

Choices for GE 4 (choose one)

1. Industrial Mathematics
2. Statistical Techniques
3. Modeling and Simulation

Details of courses under B.Sc. (Hons.) Applied Mathematics

Course	***Credits**	
	Theory + Practical	Theory + Tutorial
I. Core Course (14 Papers)	14×4 = 56	14×5 = 70
Core Course Practical / Tutorial* (14 Papers)	14×2 = 28	14×1 = 14
II. Elective Course (8 Papers)		
A.1. Discipline Specific Elective **(4 Papers)**	4×4 = 16	4×5 = 20
A.2. Discipline Specific Elective Practical/ Tutorial* **(4 Papers)**	4×2 = 8	4×1 = 4
B.1. Generic Elective/ Interdisciplinary **(4 Papers)**	4×4 = 16	4×5 = 20
B.2. Generic Elective Practical/ Tutorial* **(4 Papers)**	4×2 = 8	4×1= 4

- Optional Dissertation or project work in place of one Discipline Specific Elective Paper (6 credits) in 6th Semester

III. Ability Enhancement Courses

1. **Ability Enhancement Compulsory Courses (AECC)**		
(2 Papers of 2 credit each)	2×2 = 4	2×2 = 4
Environmental Science English/MIL Communication		
2. Skill Enhancement Courses (SEC)		
(Minimum 2)	2×2 = 4	2×2 = 4
(2 Papers of 2 credit each)		
Total credit	140	140

Institute should evolve a system/policy about ECA/ General Interest/ Hobby/ Sports/ NCC/ NSS/ related courses on its own.

*** wherever there is a practical there will be no tutorial and vice-versa**

C 1.1 Calculus

Hyperbolic functions, higher order derivatives, Leibniz rule and its applications to problems of type $e^{ax+b}\sin x$, $e^{ax+b}\cos x$, $(ax+b)^n\sin x$, $(ax+b)^n\cos x$, concavity and inflection points, asymptotes, curve tracing in Cartesian coordinates, tracing in polar coordinates of standard curves, L'Hospital's rule, applications in business, economics and life sciences.

Reduction formulae, derivations and illustrations of reduction formulae of the type $\int \sin nx\, dx$, $\int \cos nx\, dx$, $\int \tan nx\, dx$, $\int \sec nx\, dx$, $\int (\log x)^n\, dx$, $\int sin^n x\, sin^m x\, dx$, volumes by slicing, disks and washers methods, volumes by cylindrical shells, parametric equations, parameterizing a curve, arc length, arc length of parametric curves, area of surface of revolution.

Techniques of sketching conics, reflection properties of conics, rotation of axes and second degree equations, classification into conics using the discriminant, polar equations of conics. Triple product, introduction to vector functions, operations with vector-valued functions, limits and continuity of vector functions, differentiation and integration of vector functions, tangent and normal components of acceleration, modeling ballistics and planetary motion, Kepler's second law.

List of Practicals (using any software)

(i) Plotting of graphs of function e^{ax+b}, $\log(ax+b)$, $1/(ax+b)$, $\sin(ax+b)$, $\cos(ax+b)$, $|ax+b|$

and be able to find the effect of a and b on the graph.

(ii) Plotting the graphs of polynomial of degree 4 and 5, the derivative graph, the second derivative graph and comparing them.

(iii) Sketching parametric curves (Eg. Trochoid, cycloid, epicycloids, hypocycloid).

(iv) Obtaining surface of revolution of curves.

(v) Tracing of conics in Cartesian coordinates/ polar coordinates.

(vi) Sketching ellipsoid, hyperboloid of one and two sheets, elliptic cone, elliptic, paraboloid, hyperbolic paraboloid using Cartesian co-ordinates.

(vii) Matrix operation (addition, multiplication, inverse, transpose).

Books Recommended

1. G. B. Thomas, Jr. and R. L. Finney, *Calculus: Calculus and Analytic Geometry*, 9th Ed., Pearson Education, India, 2005.

2. M.J. Strauss, G.L. Bradley and K.J. Smith, *Calculus*, 3rd Ed., Dorling Kindersley (India) Pvt. Ltd. (Pearson Education), Delhi, 2007.

3. H. Anton, I. Bivens and S. Davis, *Calculus,* 7th Ed., John Wiley and Sons (Asia) Pvt. Ltd.,Singapore, 2002.

4. R. Courant and F. John, *Introduction to Calculus and Analysis* (Volumes I & II), Springer-Verlag, New York, Inc., 1989.

C 1.2 Algebra

Polar representation of complex numbers, n^{th} roots of unity, De Moivre's theorem for rational indices and its applications.

Equivalence relations, Functions, Composition of functions, Invertible functions, One to one correspondence and cardinality of a set, Well-ordering property of positive integers, Division algorithm, Divisibility and Euclidean algorithm, Congruence relation between integers, Principles of Mathematical Induction, statement of Fundamental Theorem of Arithmetic.

Systems of linear equations, row reduction and echelon forms, vector equations, the matrix equation Ax=b, solution sets of linear systems, applications of linear systems, linear independence.

Introduction to linear transformations, matrix of a linear transformation, inverse of a matrix, characterizations of invertible matrices. Subspaces of R^n, dimension of subspaces of R^n and rank of a matrix, Eigen values, Eigen Vectors and Characteristic Equation of a matrix.

Books Recommended

1. Titu Andreescu and Dorin Andrica, *Complex Numbers from A to Z*, Birkhauser, 2006.

2. Edgar G. Goodaire and Michael M. Parmenter, *Discrete Mathematics with Graph Theory*, 3rd Ed., Pearson Education (Singapore) Pvt. Ltd., Indian Reprint, 2005**.**

3. David C. Lay, *Linear Algebra and its Applications,* 3rd Ed., Pearson Education Asia, Indian Reprint, 2007.

C 2.1 Real Analysis

Review of Algebraic and Order Properties of R, δ-neighborhood of a point in R, Idea of countable sets, uncountable sets and uncountability of R. Bounded above sets, Bounded below sets, Bounded Sets, Unbounded sets, Suprema and Infima, The Completeness Property of R, The Archimedean Property, Density of Rational (and Irrational) numbers in R, Intervals. Limit points of a set, Isolated points, Illustrations of Bolzano-Weierstrass theorem for sets.

Sequences, Bounded sequence, Convergent sequence, Limit of a sequence. Limit Theorems, Monotone Sequences, Monotone Convergence Theorem. Subsequences, Divergence Criteria, Monotone Subsequence Theorem (statement only), Bolzano-Weirstrass Theorem for Sequences. Cauchy sequence, Cauchy's Convergence Criterion.

Infinite series, convergence and divergence of infinite series, Cauchy Criterion, Tests for convergence: Comparison test, Limit Comparison test, Ratio Test, Cauchy's n^{th} root test, Integral test, Alternating series, Leibnitz test, Absolute and Conditional convergence.

Books Recommended

1. R.G. Bartle and D.R. Sherbert, *Introduction to Real Analysis,* 3rd Ed., John Wiley and Sons (Asia) Pvt. Ltd., Singapore, 2002.

2. Gerald G. Bilodeau , Paul R. Thie, G.E. Keough, *An Introduction to Analysis,* 2nd Ed., Jones & Bartlett, 2010.

3. Brian S. Thomson, Andrew. M. Bruckner, and Judith B. Bruckner, *Elementary Real Analysis,* Prentice Hall, 2001.

4. S.K. Berberian, *A First Course in Real Analysis*, Springer Verlag, New York, 1994.

C 2.2 Differential Equations

Differential equations and mathematical models. General, particular, explicit, implicit and singular solutions of a differential equation. Exact differential equations and integrating factors, separable equations and equations reducible to this form, linear equation and Bernoulli equations, special integrating factors and transformations.

Introduction to compartmental model, exponential decay model, lake pollution model (case study of Lake Burley Griffin), drug assimilation into the blood (case of a single cold pill, case of a course of cold pills), exponential growth of population, limited growth of population, limited growth with harvesting.

General solution of homogeneous equation of second order, principle of super position for homogeneous equation, Wronskian: its properties and applications, Linear homogeneous and non-homogeneous equations of higher order with constant coefficients, Euler's equation, method of undetermined coefficients, method of variation of parameters.

Equilibrium points, Interpretation of the phase plane, predatory-prey model and its analysis, epidemic model of influenza and its analysis, battle model and its analysis.

List of Practicals (using any software)

1. Plotting of second order solution family of differential equation.

2. Plotting of third order solution family of differential equation.

3. Growth model (exponential case only).

4. Decay model (exponential case only).

5. Lake pollution model (with constant/seasonal flow and pollution concentration).

6. Case of single cold pill and a course of cold pills.

7. Limited growth of population (with and without harvesting).

8. Predatory-prey model (basic volterra model, with density dependence, effect of DDT, two prey one predator).

9. Epidemic model of influenza (basic epidemic model, contagious for life, disease with carriers).

10. Battle model (basic battle model, jungle warfare, long range weapons).

11. Plotting of recursive sequences.

12. Study the convergence of sequences through plotting.

13. Verify Bolzano-Weierstrass theorem through plotting of sequences and hence identify convergent subsequences from the plot.

14. Study the convergence/divergence of infinite series by plotting their sequences of partial sum.

15. Cauchy's root test by plotting n^{th} roots.

16. Ratio test by plotting the ratio of n^{th} and $(n+1)^{th}$ term.

Books Recommended:

1. Belinda Barnes and Glenn R. Fulford, *Mathematical Modeling with Case Studies, A Differential Equation Approach using Maple and Matlab,* 2nd Ed., Taylor and Francis group, London and New York, 2009.

2. C.H. Edwards and D.E. Penny, *Differential Equations and Boundary Value Problems Computing and Modeling,* Pearson Education India, 2005.

3. S.L. Ross, *Differential Equations,* 3rd Ed., John Wiley and Sons, India, 2004.

4. Martha L Abell, James P Braselton, *Differential Equations with MATHEMATICA,* 3rd Ed., Elsevier Academic Press, 2004.

C 3.1 Theory of Real Functions

Limits of functions ($\epsilon - \delta$ approach), sequential criterion for limits, divergence criteria. Limit theorems, one sided limits. Infinite limits and limits at infinity. Continuous functions, sequential criterion for continuity and discontinuity. Algebra of continuous functions. Continuous functions on an interval, intermediate value theorem.

Differentiability of a function, Caratheodory's theorem, algebra of differentiable functions. Relative extrema, interior extremum theorem. Rolle's theorem, Mean value theorem, intermediate value property of derivatives, Darboux's theorem. Applications of mean value theorem and Taylor's theorem to inequalities.

Cauchy's mean value theorem. Taylor's theorem with Lagrange's form of remainder, Taylor's theorem with Cauchy's form of remainder.

Riemann integration, Riemann conditions of integrability. Riemann sum and definition of Riemann integral through Riemann sums; equivalence of two definitions; Riemann integrability of monotone and continuous functions, Properties of the Riemann integral.

Improper integrals; Convergence of Beta and Gamma functions.

Pointwise and uniform convergence of sequence of functions. Theorems on continuity, derivability and integrability of the limit function of a sequence of functions.

Limit superior and Limit inferior. Power series, radius of convergence.

Remark: Emphasis should be given on problems and applications of results.

Books Recommended

1. R.G. Bartle and D.R. Sherbert, *Introduction to Real Analysis*, John Wiley & Sons, 2003.

2. K.A. Ross, *Elementary Analysis*, The Theory of Calculus, Springer, 2004.

3. A. Mattuck, *Introduction to Analysis*, Prentice Hall, 1999.

4. S.R. Ghorpade and B.V. Limaye, *A Course in Calculus and Real Analysis*, Springer, 2006.

5. Charles G. Denlinger, *Elements of Real Analysis*, Jones and Bartlett (Student Edition),2011.

6. Tom M. Apostal, *Mathematical Analysis*, Narosa Publishing House, India, 1992.

7. S. K. Berberian, *A First Course in Real Analysis*, Springer Verlag, New York, 1994.

8. B. S. Thomson, J. B. Bruckner and A. W. Bruckner, *Elementary Real Analysis*, Prentice Hall, New Jersey, 2001.

C 3.2 Group Theory

Definition and examples of groups including permutation groups and quaternion groups (illustration through matrices), elementary properties of groups. Subgroups and examples of subgroups, centralizer, normalizer, center of a group, product of two subgroups.

Properties of cyclic groups, classification of subgroups of cyclic groups. Cycle notation for permutations, properties of permutations, even and odd permutations, alternating group, properties of cosets, Lagrange's theorem and consequences including Fermat's Little theorem.

External direct product of a finite number of groups, normal subgroups, factor groups, Cauchy's theorem for finite abelian groups.

Group homomorphisms, properties of homomorphisms, Cayley's theorem, properties of isomorphisms, First, Second and Third isomorphism theorems. Automorphism, inner automorphism, automorphism groups, automorphism groups of finite and infinite cyclic groups, applications of factor groups to automorphism groups,

Characteristic subgroups, Commutator subgroup and its properties.

Remark: Emphasis should be given on problems and applications of results.

Books Recommended

1.. John B. Fraleigh, *A First Course in Abstract Algebra*, 7th Ed., Pearson, 2002**.**

2. M. Artin, *Abstract Algebra*, 2nd Ed., Pearson, 2011.

3. I.N. Herstein, *Topics in Algebra*, 2nd Ed., John Wiley & Sons, 2006.

4. Joseph A. Gallian, *Contemporary Abstract Algebra,* 4th Ed., Narosa Publishing House, New Delhi, 1999.

5. Joseph J. Rotman, *An Introduction to the Theory of Groups*, 4th Ed., Springer Verlag, 1995.

6. David S. Dummit and Richard M. Foote, *Abstract Algebra,* 3rd Ed., John Wiley and Sons (Asia) Pvt. Ltd, Singapore, 2004.

C 3.3 PDE and Systems of ODE

Partial Differential Equations – Basic concepts and definitions, Mathematical Problems. First-Order Equations: Classification, Construction and Geometrical Interpretation. Method of Characteristics for obtaining General Solution of Quasi Linear Equations. Canonical Forms of First-order Linear Equations. Method of Separation of Variables for solving first – order partial differential equations.

Derivation of Heat equation, Wave equation and Laplace equation. Classification of second order linear equations as hyperbolic, parabolic or elliptic. Reduction of second order Linear Equations to canonical forms. The Cauchy problem, the Cauchy-Kowaleewskaya theorem, Cauchy problem of an infinite string. Initial Boundary Value Problems, Semi-Infinite String with a fixed end, Semi-Infinite String with a free end, equations with non-homogeneous boundary conditions, Non-Homogeneous Wave Equation. Method of separation of variables, Solving the Vibrating String problem, Solving the Heat Conduction problem

Systems of linear differential equations, types of linear systems, differential operators, an operator method for linear systems with constant coefficients, basic theory of linear systems in normal form, homogeneous linear systems with constant coefficients: Two equations in two unknown functions, The method of successive approximations, the Euler method, the modified Euler method, The Runge-Kutta method.

List of Practicals (using any software)

(i) Solution of Cauchy problem for first order PDE.

(ii) Finding the characteristics for the first order PDE.

(iii) Plot the integral surfaces of a given first order PDE with initial data.

(iv) Solution of wave equation $\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0$ for the following associated conditions

(a) $u(x,0) = \phi(x),\ \ u_t(x,0) = \psi(x),\quad x \in R,\ t > 0$

(b) $u(x,0) = \phi(x),\ \ u_t(x,0) = \psi(x),\ \ u(0,t) = 0,\ x \in (0,\infty),\ t > 0$

(c) $u(x,0) = \phi(x),\ \ u_t(x,0) = \psi(x),\quad u_x(0,t) = 0,\ x \in (0,\infty),\ t > 0$

(d) $u(x,0) = \phi(x),\ \ u_t(x,0) = \psi(x),\ \ u(0,t) = 0,\ \ u(1,t) = 0,\ \ 0 < x < l,\ t > 0$

(v) Solution of wave equation $\frac{\partial u}{\partial t} - k^2 \frac{\partial^2 u}{\partial x^2} = 0$ for the following associated conditions

(a) $u(x,0) = \phi(x),\ u(0,t) = a,\ u(l,t) = b,\ 0 < x < l,\ t > 0$

(b) $u(x,0) = \phi(x),\ x \in R,\ 0 < t < T$

(c) $u(x,0) = \phi(x),\ \ u(0,t) = a,\quad x \in (0,\infty),\ t \geq 0$

Books Recommended

1. Tyn Myint-U and Lokenath Debnath, *Linear Partial Differential Equations for Scientists and Engineers,* 4th Ed., Springer, Indian reprint, 2006.

2. S.L. Ross, *Differential Equations,* 3rd Ed., John Wiley and Sons, India, 2004.

3. Martha L Abell, James P Braselton, *Differential Equations with MATHEMATICA,* 3rd Ed., Elsevier Academic Press, 2004.

C 4.1 Multivariate Calculus

Use of Scientific Calculator is allowed.

Functions of several variables, limit and continuity of functions of two variables, Partial differentiation, total differentiability and differentiability, sufficient condition for differentiability.

Chain rule for one and two independent parameters, directional derivatives, the gradient, maximal and normal property of the gradient, tangent planes. Extrema of functions of two variables, method of Lagrange multipliers, constrained optimization problems, Definition of vector field, divergence and curl.

Double integration over rectangular region, double integration over non-rectangular region. Double integrals in polar co-ordinates, Triple integrals, Triple integral over a parallelepiped and solid regions volume by triple integrals, cylindrical and spherical co-ordinates. Change of variables in double integrals and triple integrals.

Line integrals, Applications of line integrals: Mass and Work. Fundamental theorem for line integrals, conservative vector fields, independence of path.

Green's theorem, surface integrals, integrals over parametrically defined surfaces. Stoke's theorem, The Divergence theorem.

Books Recommended

1. G.B. Thomas, Jr. and R.L. Finney, *Calculus: Calculus and Analytic Geometry*, 9th Ed., Pearson Education, India, 2005.

2. M.J. Strauss, G.L. Bradley and K.J. Smith, *Calculus,* 3rd Ed., Dorling Kindersley (India) Pvt. Ltd. (Pearson Education), Delhi, 2007.

3. E. Marsden, A.J. Tromba and A. Weinstein, *Basic multivariable calculus*, Springer (SIE), Indian reprint, 2005.

4. James Stewart, *Multivariable Calculus, Concepts and Contexts*, 2nd Ed., Brooks /Cole, Thomson Learning, USA, 2001.

5. W. Kosmala, *Advanced Calculus – A Friendly Approach*, Prentice Hall, 1999 / Pearson, 1999.

6. Tom M. Apostal, *Calculus-I and Calculus-II*, John Wiley & Sons, New Delhi.

C 4.2 Complex Analysis

Limits, Limits involving the point at infinity, continuity. Properties of complex numbers, regions in the complex plane, functions of complex variable, mappings. Derivatives, differentiation formulas, Cauchy-Riemann equations, sufficient conditions for differentiability.

Analytic functions, examples of analytic functions, exponential function, Logarithmic function, trigonometric function, derivatives of functions, definite integrals of functions. Contours, Contour integrals and its examples, upper bounds for moduli of contour integrals. Antiderivatives, proof of antiderivative theorem, Cauchy-Goursat theorem, Cauchy integral formula.

An extension of Cauchy integral formula, consequences of Cauchy integral formula, Liouville's theorem and the fundamental theorem of algebra. Convergence of sequences and series, Taylor series and its examples.

Laurent series and its examples, absolute and uniform convergence of power series, uniqueness of series representations of power series. Isolated singular points, residues, Cauchy's residue theorem, residue at infinity. Types of isolated singular points, residues at poles and its examples, definite integrals involving sines and cosines.

List of Practicals (using any software)

(i) Declaring a complex number e.g. $z_1 = 3+4i, z_2 = 4-7i.$ Discussing their algebra $z_1 + z_2$, $z_1 - z_2$, $z_1 * z_2$ and z_1 / z_2 and then plotting them.

(ii) Finding conjugate, modulus and phase angle of an array of complex numbers. e.g., $z = [2+3i, 4-2i, 6+11i, 2-5i]$.

(iii) Compute the integral over a straight line path between the two specified end points e.g., $\int_C f(z)dz,$ where C is the straight line path from $a + ib$ to $c + id$.

(iv) Perform contour integration e.g. $\int_C f(z)dz,$ where C is the contour given by $g(x, y) = 0$.

(v) Plotting of the complex functions like $f(z) = z, f(z) = z^3, f(z) = \left(z^4 - 1\right)^{1/4}$, etc.

(vi) Finding the residues of the complex function.

(vii) Taylor series expansion of a given function $f(z)$ around a given point z, given the number of terms in the Taylor series expansion. Hence comparing the function and its Taylor series expansion by plotting the magnitude of each. For example

(a) $f(z) = \exp(z)$ around $z = 0, n = 40$

(b) $f(z) = \exp(z^2)$ around $z = 0, n = 160,$ etc.

(viii) To perform Laurentz series expansion of a given function $f(z)$ around a given point z, e.g., $f(z) = (\sin z - 1)/z^4$ around $z = 0$, $f(z) = \cot(z)/z^4$ around $z = 0.$, etc.

(ix) Computing the Fourier series, Fourier sine series and Fourier cosine series of a function and plotting their graphs.

Books Recommended

1. James Ward Brown and Ruel V. Churchill, *Complex Variables and Applications*, 8th Ed., McGraw – Hill International Edition, 2009.

2. Joseph Bak and Donald J. Newman, *Complex Analysis,* 2nd Ed., Undergraduate Texts in Mathematics, Springer-Verlag New York, Inc., New York, 1997.

3. L.V. Ahlfors, *Complex Analysis*, McGraw Hill Book Company, 1966.

4. J.B. Conway, *Functions of Complex Variable I*, Springer Verlag, New York Inc, 1978.

5. Murray R. Spiegel, *Complex Variables*, Schaum's Outline Series, New York, 1964.

6. Reinhold Remmert, *Theory of Complex Functions*, Springer Verlag, 1991.

7. Walter Rudin, *Real and Complex Analysis*, McGraw Hill, New York, 1997.

8. George E. Shilov, *Elementary Real and Complex Analysis*, The MIT Press, Massachusetts, 1973.

9. Dennis G. Zill and Patrick D. Shanahan, *A First Course in Complex Analysis with Applications*, Jones & Bartlett, India, 2010.

10. James W. Brown and R.V. Churchill, *Complex Variables and Applications*, 8th Ed., McGraw Hill International Edition, 2009.

C 4.3 Rings and Linear Algebra

Definition and examples of rings, properties of rings, integral domains and fields, characteristic of a ring.

Ideals, ideal generated by a subset of a ring, operations on ideals, prime and maximal ideals.

Ring homomorphisms, properties of ring homomorphisms, polynomial rings over commutative rings, division algorithm, Eisenstein criterion.

Vector spaces, subspaces, algebra of subspaces, quotient spaces, linear combination of vectors, linear span, linear independence, basis and dimension, dimension of subspaces.

Linear transformations, null space, range, rank and nullity of a linear transformation, matrix representation of a linear transformation, algebra of linear transformations.

Dual spaces, dual basis, double dual, transpose of a linear transformation and its matrix in the dual basis, annihilators. Eigenspaces of a linear operator, diagonalizability, invariant subspaces and Cayley-Hamilton theorem, the minimal polynomial for a linear operator. Inner product spaces and norms.

Remark: Emphasis should be given on problems and applications of results.

Books Recommended

1. John B. Fraleigh, *A First Course in Abstract Algebra*, 7th Ed., Pearson Education India, 2003.

2. I.N. Herstein, *Topics in Algebra*, 2nd Ed., John Wiley & Sons, 2006.

3. Stephen H. Friedberg, Arnold J. Insel, Lawrence E. Spence, *Linear Algebra*, 4th Ed., Prentice-Hall of India Pvt. Ltd., New Delhi, 2004.

4. Joseph A. Gallian, *Contemporary Abstract Algebra,* 4th Ed., Narosa Publishing House, New Delhi, 1999.

5. S Lang, *Introduction to Linear Algebra*, 2nd Ed., Springer, 2005.

6. Gilbert Strang, *Linear Algebra and its Applications*, Thomson, 2007.

7. S. Kumaresan, *Linear Algebra- A Geometric Approach*, Prentice Hall of India,1999.

8. Kenneth Hoffman, Ray Alden Kunze, *Linear Algebra*, 2nd Ed., Prentice-Hall of India Pvt. Limited, 1971.

9. Michael Artin, *Algebra*, 2nd Ed., Pearson Prentice Hall, 2011.

10. Robinson, Derek John Scott., *An Introduction to Abstract Algebra*, Hindustan Book Agency, 2010.

C 5.1 Mechanics

Moment of a force about a point and an axis, couple and couple moment, Moment of a couple about a line, resultant of a force system, distributed force system, free body diagram, free body involving interior sections, general equations of equilibrium, two point equivalent loading, problems arising from structures, static indeterminacy.

Laws of Coulomb friction, application to simple and complex surface contact friction problems, transmission of power through belts, screw jack, wedge, first moment of an area and the centroid, other centers, Theorem of Pappus-Guldinus, second moments and the product of area of a plane area, transfer theorems, relation between second moments and products of area, polar moment of area, principal axes.

Conservative force field, conservation for mechanical energy, work energy equation, kinetic energy and work kinetic energy expression based on center of mass, moment of momentum equation for a single particle and a system of particles, translation and rotation of rigid bodies, Chasles' theorem, general relationship between time derivatives of a vector for different references, relationship between velocities of a particle for different references, acceleration of particle for different references.

Books Recommended

1. I.H. Shames and G. Krishna Mohan Rao, *Engineering Mechanics: Statics and Dynamics*, 4th Ed., Dorling Kindersley (India) Pvt. Ltd. (Pearson Education), Delhi, 2009.

2. R.C. Hibbeler and Ashok Gupta, *Engineering Mechanics: Statics and Dynamics*, 11th Ed., Dorling Kindersley (India) Pvt. Ltd. (Pearson Education), Delhi.

C 5.2 Numerical Methods and Programming

Algorithms, Convergence, Bisection method, False position method, Fixed point iteration method, Newton's method, Secant method, LU decomposition, Gauss-Jacobi, Gauss-Siedel and SOR iterative methods.

Lagrange and Newton interpolation: linear and higher order, finite difference operators.

Numerical differentiation: forward difference, backward difference and central difference. Integration: trapezoidal rule, Simpson's rule, Euler's method.

Note: Emphasis is to be laid on the algorithms of the above numerical methods.

List of Practicals (using any software)

(i) Calculate the sum 1/1 + 1/2 + 1/3 + 1/4 + ----------+ 1/ N.
(ii) To find the absolute value of an integer.
(iii) Enter 100 integers into an array and sort them in an ascending order.
(iv) Bisection Method.
(v) Newton Raphson Method.
(vi) Secant Method.
(vii) Regulai Falsi Method.
(viii) LU decomposition Method.
(ix) Gauss-Jacobi Method.
(x) SOR Method or Gauss-Siedel Method.
(xi) Lagrange Interpolation or Newton Interpolation.
(xii) Simpson's rule.

Note: For any of the CAS, Data types-simple data types, floating data types, character data types, arithmetic operators and operator precedence, variables and constant declarations, expressions, input/output, relational operators, logical operators and logical expressions, control statements and loop statements, Arrays should be introduced to the students.

Books Recommended

1. B. Bradie, *A Friendly Introduction to Numerical Analysis*, Pearson Education, India, 2007.
2. M.K. Jain, S.R.K. Iyengar and R. K. Jain, *Numerical Methods for Scientific and Engineering Computation*, 5th Ed.,New Age International Publisher, India, 2007.
3. C.F. Gerald and P.O. Wheatley, *Applied Numerical Analysis*, Pearson Education, India,7th Ed., 2008.

C 6.1 Integral Equations and Calculus of Variation

Preliminary Concepts: Definition and classification of linear integral equations. Conversion of initial and boundary value problems into integral equations. Conversion of integral equations into differential equations. Integro-differential equations.

Fredholm Integral Equations: Solution of integral equations with separable kernels, Eigen values and Eigen functions. Solution by the successive approximations, Neumann series and resolvent kernel. Solution of integral equations with symmetric kernels, Hilbert-Schmidt theorem, Green's function approach.

Classical Fredholm Theory: Fredholm method of solution and Fredholm theorems.

Volterra Integral Equations: Successive approximations, Neumann series and resolvent kernel. Equations with convolution type kernels.

Solution of integral equations by transform methods: Singular integral equations, Hilbert-transform, Cauchy type integral equations.

Calculus of Variations: Basic concepts of the calculus of variations such as functionals, extremum, variations, function spaces, the brachistochrone problem.

Necessary condition for an extremum, Euler`s equation with the cases of one variable and several variables, Variational derivative. Invariance of Euler`sequations. Variational problem in parametric form.

General Variation: Functionals dependent on one or two functions, Derivation of basic formula, Variational problems with moving boundaries, Broken extremals: Weierstrass–Erdmann conditions.

Books Recommended

1. Abdul J. Jerry, *Introduction to Integral Equations with Applications*, 2nd Ed., Clarkson University Wiley Publishers, 1999.
2. Chambers, Ll. G., *Integral Equations: A short Course*, International Text Book Company Ltd., 1976.
3. R. P. Kanwal, *Linear Integral Equations*, 2nd Ed., Birkhauser Bosten, 1997.
4. Hochstadt Harry, *Integral Equations*, John Wiley & Sons, 1989.
5. I. M. Gelfand, S.V. Fomin, *Calculus of Variations*, Dover Books, 2000.
6. Weinstock Robert, *Calculus of Variations with Applications to Physics and Engineering*, Dover Publications, INC., 1974.

C 6.2 Laplace Transform

Laplace Transform: Laplace of some standard functions, Existence conditions for the Laplace Transform, Shifting theorems, Laplace transform of derivatives and integrals, Inverse Laplace transform and their properties, Convolution theorem, Initial and final value theorem, Laplace transform of periodic functions, error functions, Heaviside unit step function and Dirac delta function, Applications of Laplace transform to solve ODEs and PDEs.

Finite Laplace Transform: Definition and properties, Shifting and scaling theorem.

Z-Transform: Z–transform and inverse Z-transform of elementary functions, Shifting theorems, Convolution theorem, Initial and final value theorem, Application of Z-transforms to solve difference equations.

Hankel Transform: Basic properties of Hankel Transform, Hankel Transform of derivatives, Application of Hankel transform to PDE.

Mellin Transform: Definition and properties of Mellin transform, Shifting and scaling properties, Mellin transforms of derivatives and integrals, Applications of Mellin transform.

Fourier series: Trigonometric Fourier series and its convergence.Fourier series of even and odd functions, Gibbs phenomenon, Fourier half-range series, Parseval`s identity, Complex form of Fourier series.

Fourier Transforms: Fourier integrals, Fourier sine and cosine integrals, Complex form of Fourier integral representation, Fourier transform, Fourier transform of derivatives and integrals, Fourier sine and cosine transforms and their properties, Convolution theorem, Application of Fourier transforms to Boundary Value Problems.

Books Recommended

1. E. Kreyszig, *Advanced Engineering Mathematics*, John Wiley & Sons, 2011.
2. R.K. Jain and S.R.K. Iyenger, *Advanced Engineering Mathematics*, Narosa Publishing House, 2009.
3. F. B. Hildebrand, *Methods of Applied Mathematics*, Courier Dover Publications, 1992.
4. L. Debanth and D. Bhatta, *Integral Transforms and Their Applications*, 2nd Ed., Taylor and Francis Group, 2007.

DSE 1.1 Number Theory

Linear Diophantine equation, prime counting function, statement of prime number theorem, Goldbach conjecture, linear congruences, complete set of residues, Chinese Remainder theorem, Fermat's Little theorem, Wilson's theorem.

Number theoretic functions, sum and number of divisors, totally multiplicative functions, definition and properties of the Dirichlet product, the Möbius Inversion formula, the greatest integer function, Euler's phi-function, Euler's theorem, reduced set of residues, some properties of Euler's phi-function.

Order of an integer modulo n, primitive roots for primes, composite numbers having primitive roots, Euler's criterion, the Legendre symbol and its properties, quadratic reciprocity, quadratic congruences with composite moduli. Public key encryption, RSA encryption and decryption, the equation $x^2 + y^2 = z^2$, Fermat's Last theorem.

Books Recommended

1. David M. Burton, *Elementary Number Theory,* 6th Ed., Tata McGraw-Hill Edition, Indian reprint, 2007.

2. Neville Robinns, *Beginning Number Theory*, 2nd Ed., Narosa Publishing House Pvt. Limited, Delhi, 2007.

DSE 1.2 Graph Theory

Definition, examples and basic properties of graphs, pseudographs, complete graphs, bi-partite graphs, isomorphism of graphs, paths and circuits, Eulerian circuits, Hamiltonian cycles, the adjacency matrix, weighted graph, travelling salesman's problem, shortest path, Dijkstra's algorithm, Floyd-Warshall algorithm.

Applications of paths and circuits: the Chinese postman problem, digraphs, the Bellman-Ford algorithm, tournaments, directed network, scheduling problems, definition, examples and basic properties of trees, spanning trees, minimum spanning tree algorithms, Kruskal's algorithm, Prim's algorithm, acyclic digraphs, Bellman's algorithm.

Planar graphs, colouring of graphs, statement of the four-colour theorem, the five colour theorem, circuit testing, facilities design, flows and cuts, construction of flows, constructing maximal flows, rational weights, applications of directed networks, matchings.

Books Recommended

1. Edgar G. Goodaire and Michael M. Parmenter, *Discrete Mathematics with Graph Theory,* 2nd Ed., Pearson Education (Singapore) P. Ltd., Indian Reprint, 2003.
2. Rudolf Lidl and Günter Pilz, *Applied Abstract Algebra*, 2nd Ed., Undergraduate Texts in Mathematics, Springer (SIE), Indian reprint, 2004.
3. C.L. Liu, *Elements of Discrete Mathematics,* 2nd Ed., Tata McGraw Hill Publishing Company Ltd., 2001.

DSE 1.3 Linear Programming

Linear Programming Problems, Graphical Approach for Solving some Linear Programs. Convex Sets, Supporting and Separating Hyperplanes. Theory of simplex method, optimality and unboundedness, the simplex algorithm, simplex method in tableau format, introduction to artificial variables, two-phase method, Big-M method and their comparison.

Duality, formulation of the dual problem, primal-dual relationships, economic interpretation of the dual, sensitivity analysis.

Books Recommended

1. Mokhtar S. Bazaraa, John J. Jarvis and Hanif D. Sherali, *Linear programming and Network Flows,* 2nd Ed., John Wiley and Sons, India, 2004.

2. F. S. Hillier and G. J. Lieberman, *Introduction to Operations Research*, 8th Ed., Tata McGraw Hill, Singapore, 2004.

3. Hamdy A. Taha, *Operations Research, An Introduction,* 8th Ed., Prentice-Hall India, 2006.

DSE 2.1 Control Theory

Mathematical models of control systems, State space representation, Autonomous and non autonomous systems, State transition matrix, Peano series solution of linear dynamical system.

Block diagram, Transfer function, Realization, Controllability, Kalman theorem, Controllability Grammian, Control computation using Grammian matrix, Observability, Duality theorems.

Discrete control systems, Controllability and Observability results for discrete systems.

Companion form, Feedback control, State observer, Realization Liapunov stability, Stability analysis for linear systems, Liapunov theorems for stability and instability for nonlinear systems, Stability analysis through Linearization, Routh criterion, Nyquist criterion, Stabilizability and detachability, State feedback of multivariable system, Riccatti equation, Calculus of variation, Euler-Hamiltonian equations, Optimal control for nonlinear control systems, Computation of optimal control for linear systems.

Control systems on Hilbert spaces, Semi group theory, Mild solution, Control of a linear system.

Books Recommended

1. S. Barnett, *Introduction to Mathematical Control theory,* Clarendon Press Oxford, 1975.
2. R.V. Dukkipati, *Control Systems*, Narosa Publication House Pvt. Ltd., 2005.
3. I.J. Nagrath and M. Gopal, *Control System Engineering*, New Age International, 2001.
4. B. Datta, *Numerical Methods for Linear Control Systems*, Academic Press Elsevier, 2005.
5. B.C. Kho , *Automatic Control System*, Prentice Hall, 2001.

DSE 2.2 Approximation Theory

Concept of best approximation in a normed linear space, Existence of the best approximation, Uniqueness problem, Convexity-uniform convexity, strict convexity and their relations, Continuity of the best approximation operator.

The Weierstrass theorem, Bernstein polynomials, Korovkin theorem, Algebraic and trigonometric polynomials of the best approximation, Lipschitz class, Modulus of continuity, Integral modulus of continuity and their properties.

Bernstein's inequality, Jackson's theorems and their converse theorems, Approximation by means of Fourier series.

Positive linear operators, Monotone operators, Simultaneous approximation, $p\,L$ -approximation, Approximation of analytic functions.

Books Recommended

1. E.W. Cheney, *Introduction to Approximation Theory*, AMS Chelsea Publishing Co., 1981.

2. G.G. Lorentz, *Bernstein Polynomials*, Chelsea Publishing Co., 1986.

3. I.P. Natanson, *Constructive Function Theory Volume-I*, Fredrick Ungar Publishing Co., 1964.

4. H.M. Mhaskar and D.V. Pai, *Fundamentals of Approximation Theory*, Narosa Publishing House, 2000.

5. A.F. Timan, *Theory of Approximation of Functions of a Real Variable*, Dover Publication Inc., 1994.

DSE 2.3 Combinatorial Optimization

Introduction: Optimization problems, neighborhoods, local and global optima, convex sets and functions, simplex method, degeneracy, duality and dual simplex algorithm, computational considerations for the simplex and dual simplex algorithms-Dantzig-Wolfe algorithms.

Integer Linear Programming: Cutting plane algorithms, branch and bound technique and approximation algorithms for traveling salesman problem.

Graph Algorithms: Primal-Dual algorithm and its application to shortest path, Math-flow problems (Ford and Fulkerson labeling algorithms, Dijkstra's algorithm, Ford-Warshall algorithms), networking labeling and digraph search, Max-flow problem, matching problem, bipartite matching algorithm, non-bipartite matching algorithms, weighted matching-hungarian method for the assignment problem, non-bipartite weighted matching problem, efficient spanning tree algorithms, algorithm for matroid intersection problem.

Books Recommended

1. C.H. Papadimitriou and K. Steiglitz, *Combinatorial Optimization: Algorithms and Complexity*, Prentice-Hall of India, 2006.

2. K. Lange, *Optimization*, Springer, 2004.

3. Mokhtar S. Bazaraa, John J. Jarvis and Hanif D. Sherali, *Linear Programming and Network Flows*, John Wiley & Sons, 2004.

4. H.A. Taha, *Operations Research: An Introduction*, 8th Ed., Prentice Hall, 2006.

DSE 3.1 Mathematical Modeling

Power series solution of a differential equation about an ordinary point, solution about a regular singular point, Bessel's equation and Legendre's equation, Laplace transform and inverse transform, application to initial value problem up to second order.

Monte Carlo Simulation Modeling: simulating deterministic behavior (area under a curve, volume under a surface), Generating Random Numbers: middle square method, linear congruence, Queuing Models: harbor system, morning rush hour, Overview of optimization modeling, Linear Programming Model: geometric solution algebraic solution, simplex method, sensitivity analysis.

List of Practicals (using any software)

(i) Plotting of Legendre polynomial for n = 1 to 5 in the interval [0,1]. Verifying graphically that all the roots of $P_n(x)$ lie in the interval [0,1].
(ii) Automatic computation of coefficients in the series solution near ordinary points
(iii) Plotting of the Bessel's function of first kind of order 0 to 3.
(iv) Automating the Frobenius Series Method
(v) Random number generation and then use it for one of the following (a) Simulate area under a curve (b) Simulate volume under a surface
(vi) Programming of either one of the queuing model (a) Single server queue (e.g. Harbor system) (b) Multiple server queue (e.g. Rush hour)
(vii) Programming of the Simplex method for 2/3 variables.

Books Recommended

1. Tyn Myint-U and Lokenath Debnath, *Linear Partial Differential Equation for Scientists and Engineers*, Springer, Indian reprint, 2006.

2. Frank R. Giordano, Maurice D. Weir and William P. Fox, *A First Course in Mathematical Modeling*, Thomson Learning, London and New York, 2003.

DSE 3.2 Coding Theory

The communication channel, The coding problem, Block codes, Hamming metric, Nearest neighbour decoding, Linear codes, Generator and Parity-check matrices, Dual code, Standard array decoding, Syndrome decoding.

Hamming codes, Golay codes, Reed-Muller codes, Codes derived from Hadamard matrices.
Bounds on codes: Sphere packing bound, Perfect codes, Gilbert-Varshamov bound, Singleton bound, MDS codes, Plotkin bound.

Weight distributions of codes, Mac Williams identities. Algebra of polynomials, Residue class rings, Finite fields, Cyclic codes, Generator polynomial and check polynomial, Defining set of a cyclic code.

BCH bound, Encoding and decoding of cyclic codes, Hamming and Golay codes as cyclic codes, BCH codes, Reed-Solomon codes, Quadratic residue codes, Graphical codes, Convolutional codes.

Books Recommended

1. F.J. Mac Williams and N.J.A.Sloane, *The Theory of Error Correcting Codes*, North Holland, 1977.

2. S. Ling and C. Xing, *Coding Theory: A First Course*, Cambridge University Press, 2004.

3. R.M. Roth, *Introduction to Coding Theory*, Cambridge University Press, 2006.

4. V. Pless, *Introduction to The Theory of Error Correcting Codes,* 3rd Ed., John Wiley, 1999.

5. W.C. Huffman, and V. Pless, *Fundamentals of Error Correcting Codes*, Cambridge University Press, 2003.

6. J. H. van Lint, *Introduction to Coding Theory*, 3rd Ed., Springer, 1998.

7. T. K. Moon, *Error Correction Coding*, John Wiley and Sons, 2005.

DSE 3.3 Wavelet Theory

Review of basic concepts and theorems of Functional analysis and Lebesgue theory.

Advanced Fourier Analysis: Fourier transform (F.T.) of functions in $L_1(R)$. Basic properties of F.T. of functions in $L_\infty(R)$. Inverse Fourier transform, Convolution, Approximate identity. Auto correlation of functions in $L_2(R)$, F.T. of functions in $L_1(R) \cap L_2(R)$. Various versions of Parseval's identity (P.I.) of functions in $L_1(R) \cap L_2(R)$. Evaluation of improper integrals using P.I., Plancheral theorem.

Trigonometric Fourier Series (TFS) of functions of L1[0, 2π] and its complex form. Dirichlet conditions, Gibbs phenomenon, modulus of continuity, integral modulus of continuity. Convergence of TFS in $L_1[0, 2\pi]$, Bessel`s inequality for functions of $L_2[0, 2\pi]$. Summability of TFS. The Poisson`s summation formula and its applications.

Time Frequency Analysis: Window functions and their examples. Windowed functions. The Gabor transform STFS, the uncertainty principal, the classical Shannon sampling theorem, frames, exact and tight frames.

Wavelet Transform: Isometric isomorphism between L_2 and $L_2[0, 2\pi]$, wavelet transform, wavelet series. Basic wavelets (Haar/Shannon/Daubechies), integral wavelet, orthogonal wavelets, multi-resolution analysis, reconstruction of wavelets and applications.

Books Recommended

1. C.K. Chui, *Introduction to Wavelet*, Academic Press, 1992.

2. G. Narici, L. Bachman, E. Beckenstein, *Fourier and Wavelet Analysis*, Springer, 2005.

3. A.K. Chan, Chens Peng, *Wavelet for Sensing Technology*, 2003.

4. I. Daubechies, , *Ten Lectures in Wavelets*, SIAM, 1992.

5. T.H. Koorniwinder, *Wavelet: An Elementary Treatment of Theory and Applications*, World Scientific Publication, 2009.

DSE 4.1 Bio-Mathematics

Mathematical Biology and the modeling process: an overview. Continuous models: Malthus model, logistic growth, Allee effect, Gompertz growth, Michaelis-Menten Kinetics, Holling type growth, Bacterial growth in a Chemostat, Harvesting a single natural population, Prey predator systems and Lotka-Volterra equations, Populations in competitions, Epidemic Models (SI, SIR, SIRS, SIC), Activator-Inhibitor system, Insect Outbreak Model: Spruce Budworm, Numerical solution of the models and its graphical representation. Qualitative analysis of continuous models: Steady state solutions, stability and linearization, multiple species communities and Routh-Hurwitz Criteria, Phase plane methods and qualitative solutions, bifurcations and limit cycles with examples in the context of biological scenario. Spatial Models: One species model with diffusion, Two species model with diffusion, Conditions for diffusive instability, Spreading colonies of microorganisms, Blood flow in circulatory system, Travelling wave solutions, Spread of genes in a population. Discrete Models: Overview of difference equations, steady state solution and linear stability analysis, Introduction to Discrete Models, Linear Models, Growth models, Decay models, Drug Delivery Problem, Discrete Prey-Predator models, Density dependent growth models with harvesting, Host-Parasitoid systems (Nicholson-Bailey model), Numerical solution of the models and its graphical representation. Case Studies: Optimal Exploitation models, Models in Genetics, Stage Structure Models, Age Structure Models.

Books Recommended

1. L.E. Keshet, *Mathematical Models in Biology*, SIAM, 1988.

2. J.D., Murray, *Mathematical Biology*, Springer, 1993.

3. Y.C. Fung, *Biomechanics*, Springer-Verlag, 1990.

4. F. Brauer, P.V.D. Driessche, and J. Wu, *Mathematical Epidemiology*, Springer, 2008.

5. M. Kot, *Elements of Mathematical Ecology*, Cambridge University Press, 2001.

DSE 4.2 Stochastic Processes

Introduction to stochastic processes, Poisson Process: Inter arrival and waiting time distributions, conditional distributions of the arrival times, non-homogeneous Poisson process, compound Poisson random variables and Poisson processes, conditional Poisson processes.

Markov Chains: Introduction and examples, Chapman-Kolmogorov equations and classification of states, limit theorems, transitions among classes, the Gambler's ruin problem, mean time in transient states, branching processes, applications of Markov chain, time reversible Markov chains, semi Markov processes.

Continuous-Time Markov Chains: Introduction, continuous time Markov chains, birth and death processes, The Kolmogorov differential equations, limiting probabilities, time reversibility, applications of reversed chain to queueing theory.

Martingales: Introduction, stopping times, Azuma's inequality for martingales, submartingales, supermartingles, martingale convergence theorem.

Brownian Motion and other Markov Processes: Introduction, hitting time, maximum variable, Arc sine laws, variations on Brownian motion, Brownian motion with drift, backward and forward diffusion equations.

Books Recommended

1. S. M. Ross, *Stochastic Processes*, Wiley India Pvt. Ltd., 2nd Ed., 2008.

2. Z. Brzezniak and T. Zastawniak, *Basic Stochastic Processes: A Course through Exercises*, Springer, 1992.

3. J. Medhi, *Stochastic Processes*, New Age Science, 2009.

4. S.I. Resnick, *Adventures in Stochastic Processes*, Birkhauser, 1999.

5. P.G. Hoel and C.J. Stone, *Introduction to Stochastic Processes*, Waveland Press, 1986.

DSE 4.3 Difference Equations

Difference Calculus: Introduction, The Difference Operator, Summation, Generating Functions and Approximate Summation.

Linear Difference Equations: First Order Equations, General Results for Linear Equations, Solving Linear Equations, Applications, Equations with Variable Coefficients, Nonlinear Equations that can be Linearized, the z-Transform.

Stability Theory: Initial Value Problems for Linear Systems, Stability of Linear Systems, Phase Plane Analysis for Linear Systems, Fundamental Matrices and Floquet Theory, Stability of Nonlinear Systems, Chaotic Behavior.

Asymptotic Methods: Introduction, Asymptotic Analysis of Sums, Linear Equations, Nonlinear Equations.

Books Recommended

1. Walter Kelley and Allan Peterson, *Difference Equations, An Introduction with Applications*, Academic Press, 1991.

2. Calvin Ahlbrant and Allan Peterson, *Discrete Hamiltonian Systems, Difference Equations, Continued Fractions and Riccati Equations*, Kluwer, 1996.

3. Saber Elaydi, *An Introduction to Difference Equations*, Springer, 1999.

SEC 1.1 Electronic Commerce

Building Blocks of Electronic Commerce: Introduction, internet and networking technologies, Internet and network protocols, web server scalability, software technologies for building E-commerce applications, distributed objects, object request brokers, component technology, web services, web application architectures, design of auctions, optimization algorithms for market places, multi-agent systems. Global e-Commerce and Law: Cyber law in India. Comparative evaluation of Cyber laws of certain countries.

Books Recommended

1. E.M. Awad, *Electronic Commerce from Vision to Fulfillment* (3rd ed.), Prentice-Hall of India, 2006.

2. P.T. Joseph, *E-Commerce: An Indian Perspective*, Prentice-Hall of India, 2007.

3. Scott Bonneau, Tammy Kohl, Jeni Tennison, Jon Duckett and Kevin Williams, *XML Design Handbook*, Wrox Press Ltd., 2003.

4. Michael Cheshar, Ricky Kaura, and Peter Linton, *Electronic Business and Commerce*, Springer, 2003.

5. W.J. Pardi, *XML in Action: Learn to quickly create dynamic, data-driven sites with the Web's hottest new technology*, Prentice Hall of India, 1999.

6. P. Weill and M.R. Vitale, *Place to Space: Migrating to eBusiness Models*, Harvard Business School Press, 2001.

7. D. Whiteley, *eCommerce: Strategy, Technologies and Applications*, Tata McGraw-Hill Edition, 2001.

8. M. Fitzgerald, *Building B2B Applications with XML: A Resource Guide*, John Wiley and Sons, Inc., 2001.

SEC 1.2 Computer Graphics

Development of computer Graphics: Raster Scan and Random Scan graphics storages, displays processors and character generators, colour display techniques, interactive input/output devices. Points, lines and curves: Scan conversion, line-drawing algorithms, circle and ellipse generation, conic-section generation, polygon filling anti aliasing. Two-dimensional viewing: Coordinate systems, linear transformations, line and polygon clipping algorithms.

Books Recommended

1. D. Hearn and M.P. Baker, *Computer Graphics*, 2nd Ed., Prentice–Hall of India, 2004.

2. J.D. Foley, A van Dam, S.K. Feiner and J.F. Hughes, *Computer Graphics: Principals and Practices*, 2nd Ed., Addison-Wesley, MA, 1990.

3. D.F. Rogers, *Procedural Elements in Computer Graphics*, 2nd Ed., McGraw Hill Book Company, 2001.

4. D.F. Rogers and A.J. Admas, *Mathematical Elements in Computer Graphics*, 2nd Ed., McGraw Hill Book Company, 1990.

SEC 1.3 Operating Systems

Introduction: Operating System as a resource manager, operating system classification, system calls, traps, architectures for operating systems.

Device Management: Goals of I/O software, Design of device drivers. Processor Management: Process overview, process states and state transition, multiprogramming, multi-tasking, levels of schedulers and scheduling algorithms. Process Synchronization - Critical section and mutual exclusion problem, classical synchronization problems, deadlock prevention. Multithreading Memory Management: Classical memory management techniques, paging, segmentation, virtual memory.

File Management: Overview of file management system, disk space management, directory structures. Protection domains, access control lists, protection models.

Books Recommended:

1. A.S. Tanenbaum, *Modern Operating Systems*, 3rd Ed., Prentice-Hall of India, 2008.

2. William Stallings, *Operating Systems: Internals and Design Principles*, 5th Ed., Prentice-Hall of India, 2006.

3. Gary Nutt, *Operating Systems: A Modern Approach*, 3rd Ed., Addison Wesley, 2004.

4. D.M. Dhamdhere, *Operating Systems: A Concept Based Approach*, 2nd Ed., Tata McGraw-Hill, 2007.

SEC 2.1 Latex and Web Designing

LaTeX: Elements of LaTeX, typesetting mathematics, graphics in LaTeX, PSTricks, Beamer presentation

HTML: HTML basics, creating simple web pages, images and links, design of web pages, CSS, MathJaX

Books Recommended

1. Martin J. Erickson and Donald Bindner, *A Student's Guide to the Study, Practice, and Tools of Modern Mathematics*, CRC Press, Boca Raton, FL, 2011.

2. L. Lamport. LATEX: *A Document Preparation System, User's Guide and Reference Manual*, Addison-Wesley, New York, second edition, 1994.

SEC 2.2 Transportation and Game Theory

Transportation problem and its mathematical formulation, northwest-corner method least cost method and Vogel approximation method for determination of starting basic solution, algorithm for solving transportation problem, assignment problem and its mathematical formulation, Hungarian method for solving assignment problem.

Game theory: formulation of two person zero sum games, solving two person zero sum games, games with mixed strategies, graphical solution procedure.

Books Recommended

1. Mokhtar S. Bazaraa, John J. Jarvis and Hanif D. Sherali, *Linear Programming and Network Flows*, 2nd Ed., John Wiley and Sons, India, 2004.

2. F. S. Hillier and G. J. Lieberman, *Introduction to Operations Research*, 9th Ed., Tata McGraw Hill, Singapore, 2009.

3. Hamdy A. Taha, *Operations Research, An Introduction*, 8th Ed., Prentice-Hall India, 2006.

SEC 2.3 Fuzzy Sets and Logics

Fuzzy Sets and Uncertainty: Uncertainty and information, fuzzy sets and membership functions, chance versus fuzziness, properties of fuzzy sets, fuzzy set operations.

Fuzzy Relations: Cardinality, operations, properties, fuzzy Cartesian product and composition, fuzzy tolerance and equivalence relations, forms of composition operation.

Fuzzification and Defuzzification: Various forms of membership functions, fuzzification, defuzzification to crisp sets and scalars.

Fuzzy Logic and Fuzzy Systems: Classic and fuzzy logic, approximate reasoning, Natural language, linguistic hedges, fuzzy rule based systems, graphical technique of inference.

Development of membership functions: Membership value assignments: intuition, inference, rank ordering, neural networks, genetic algorithms, inductive reasoning.

Fuzzy Arithmetic and Extension Principle: Functions of fuzzy sets, extension principle, fuzzy mapping, interval analysis, vertex method and DSW algorithm.

Fuzzy Optimization: One dimensional fuzzy optimization, fuzzy concept variables and casual relations, fuzzy cognitive maps, agent based models.

Fuzzy Control Systems: Fuzzy control system design problem, fuzzy engineering process control, fuzzy statistical process control, industrial applications.

Books Recommended

1 T.J. Ross, *Fuzzy Logic with Engineering Applications*, 3rd Ed., Wiley India Pvt. Ltd., 2011.

2 H.J. Zimmerman, *Fuzzy Set Theory and its Application*, 3rd Ed., Springer India Pvt. Ltd., 2006.

3 G. Klir and B. Yuan, *Fuzzy Set and Fuzzy Logic: Theory and Applications*, Prentice Hall of India Pvt. Ltd., 2002.

4 G. Klir and T. Folger, *Fuzzy Sets, Uncertainty and Information*, Prentice Hall of India Pvt. Ltd., 2002.

GE 1.1 Object Oriented Programming in C++

OOP Paradigm: Comparison of Programming paradigms, Characteristics of Object-Oriented Programming Languages, Object-based programming languages C++: Brief History of C++, Structure of a C++ program, Difference between C and C++ - cin, cout, new, delete operators, ANSI/ISO Standard C++, Comments, Working with Variables and constant Qualifiers. Enumeration, Arrays and Pointer. Implementing oops concepts in C++ Objects, Classes, Encapsulation, Data Abstraction, Inheritance, Polymorphism, Dynamic Binding, Message Passing, Default Parameter Value, Using Reference variables with Functions.

Abstract data types, Class Component, Object & Class, Constructors Default and Copy Constructor, Assignment operator deep and shallow coping, Access modifiers – private, public and protected. Implementing Class Functions within Class declaration or outside the Class declaration, instantiation of objects, Scope resolution operator, Working with Friend Functions, sing Static Class members. Understanding Compile Time Polymorphism function overloading

Rules of Operator Overloading (Unary and Binary) as member function/friend function, Implementation of operator overloading of Arithmetic Operators, Overloading Output/ Input, Prefix/ Postfix Increment and decrement Operators, Overloading comparison operators, Assignment, subscript and function call Operator , concepts of name spaces.

Practical to be performed in lab.

Books Recommended

1. A. R. Venugopal, Rajkumar, and T. Ravishanker, M*astering C++*, TMH, 1997.

2. S. B. Lippman and J. Lajoie, *C++ Primer,* 3rd Ed., Addison Wesley, 2000.

3. Bruce Eckel, *Thinking in C++*, 2nd Ed., President, Mindview Inc., Prentice Hall.

4. D. Parasons, *Object Oriented Programming with C++*, BPB Publication.

5. Bjarne Stroustrup , *The C++ Programming Language*, 3rd Ed., Addison Welsley.

6. Steven C. Lawlor, *The Art of Programming Computer Science with C++*, Vikas Publication.

7. Schildt Herbert, *C++: The Complete Reference*, 4th Ed., Tata McGraw Hill, 1999.

GE 1.2 Finite Element Methods

Introduction to finite element methods, comparison with finite difference methods. Methods of weighted residuals, collocations, least squares and Galerkin's method. Variational formulation of boundary value problems, equivalence of Galerkin and Ritz methods.

Applications to solving simple problems of ordinary differential equations.

Linear, quadratic and higher order elements in one dimensional and assembly, solution of assembled system.

Simplex elements in two and three dimensions, quadratic triangular elements, rectangular elements, serendipity elements and isoperimetric elements and their assembly, discretization with curved boundaries Interpolation functions, numerical integration, and modeling considerations**.** Solution of two dimensional partial differential equations under different Geometric conditions.

Books Recommended

1. J.N. Reddy, *Introduction to the Finite Element Methods*, Tata McGraw-Hill, 2003.

2. K.J. Bathe, *Finite Element Procedures*, Prentice-Hall, 2001.

3. R.D. Cook, D.S. Malkus and M.E. Plesha, *Concepts and Applications of Finite Element Analysis*, John Wiley, 2002.

4. J.R. Hughes Thomas, *The Finite Element Method: Linear Static and Dynamic Finite Element Analysis*, 2000.

5. George R. Buchanan, *Finite Element Analysis*, 1994.

GE 2.1 Mathematical Finance

Basic principles: Comparison, arbitrage and risk aversion, Interest (simple and compound, discrete and continuous), time value of money, inflation, net present value, internal rate of return (calculation by bisection and Newton-Raphson methods), comparison of NPV and IRR Bonds, bond prices and yields, Macaulay and modified duration, term structure of interest rates: spot and forward rates, explanations of term structure, running present value, floating-rate bonds, immunization, convexity, putable and callable bonds.

Asset return, short selling, portfolio return, (brief introduction to expectation, variance, covariance and correlation), random returns, portfolio mean return and variance, diversification, portfolio diagram, feasible set, Markowitz model (review of Lagrange multipliers for 1 and 2 constraints), Two fund theorem, risk free assets, One fund theorem, capital market line, Sharpe index. Capital Asset Pricing Model (CAPM), betas of stocks and portfolios, security market line, use of CAPM in investment analysis and as a pricing formula, Jensen's index.

Books Recommended

1. David G. Luenberger, *Investment Science*, Oxford University Press, Delhi, 1998.

2. John C. Hull, Options, *Futures and Other Derivatives*, 6th Ed., Prentice-Hall India, Indian reprint, 2006.

3. Sheldon Ross, *An Elementary Introduction to Mathematical Finance*, 2nd Ed., Cambridge University Press, USA, 2003.

GE 2.2 Econometrics

Statistical Concepts Normal distribution; chi-square, t- and F-distributions; estimation of parameters; properties of estimators; testing of hypotheses: defining statistical hypotheses; distributions of test statistics; testing hypotheses related to population parameters; Type I and Type II errors; power of a test; tests for comparing parameters from two samples.

Simple Linear Regression Model: Two Variable Case Estimation of model by method of ordinary least squares; properties of estimators; goodness of fit; tests of hypotheses; scaling and units of measurement; confidence intervals; Gauss-Markov theorem; forecasting.

Multiple Linear Regression Model Estimation of parameters; properties of OLS estimators; goodness of fit - R2 and adjusted R2; partial regression coefficients; testing hypotheses – individual and joint; functional forms of regression models; qualitative (dummy) independent variables.

Violations of Classical Assumptions: Consequences, Detection and Remedies Multicollinearity; heteroscedasticity; serial correlation.

Specification Analysis Omission of a relevant variable; inclusion of irrelevant variable; tests of specification errors.

Books Recommended

1. Jay L. Devore, *Probability and Statistics for Engineers*, Cengage Learning, 2010.

2. John E. Freund, *Mathematical Statistics*, Prentice Hall, 1992.

3. Richard J. Larsen and Morris L. Marx, *An Introduction to Mathematical Statistics and its Applications*, Prentice Hall, 2011.

4. D.N. Gujarati and D.C. Porter, *Essentials of Econometrics*, 4th Ed., McGraw Hill, International Edition, 2009.

5. Christopher Dougherty, *Introduction to Econometrics,* 3rd Ed., Oxford University Press, Indian edition, 2007.

GE 3.1 Digital Signal Processing

Discrete time signals and systems, Z-transforms, structures for digital filters, design procedures for FIR and IIR filters. Frequency transformations: linear phase design; DFT. Methods for computing FFT. Noise analysis of digital filters, power spectrum estimation. Signals and Signal Processing: characterization and classification of signals, typical Signal Processing operations, example of typical Signals, typical Signals Processing applications. Time Domain Representation of Signals and Systems: Discrete Time Signals, Operations on Sequences, the sampling process, Discrete-Time systems, Time-Domain characterization of LTI Discrete-Time systems.

Transform-Domain Representation of Signals: the Discrete-Time Fourier Transform, Discrete Fourier Transform, DFT properties, computation of the DFT of real sequences, Linear Convolution using the DFT. Z-transforms, Inverse z-transform, properties of z transform, transform domain representations of random signals, FFT. Transform-Domain Representation of LTI Systems: the frequency response, the transfer function, types of transfer function, minimum-phase and maximum-phase transfer functions.

Digital Processing of continuous-time signals: sampling of continuous signals, analog filter design, anti-aliasing, filter design, sample-and-hold circuits, A/D and D/A converter, reconstruction filter design. Digital Filter Structure: Block Diagram representation, Signal Flow Graph Representation, Equivalent Structures, FIR Digital Filter Structures, IIR Filter Structures. transfer, modes of data transfer, priority interrupt, direct memory access, input-output processor.

Digital Filter Design: Impulse invariance method of IIR filter design, Bilinear Transform method Of IIR Filter Design, Design of Digital IIR notch filters, FIR filter Design based on truncated Fonnerfonnersens, FIR filter design based on Frequency Sampling approach. Applications of DSP.

Books Recommended

1. Sanjit K. Mitra, *Digital Signal Processing a Computer based approach*, TMH, 2009.

2. Allan Y. Oppenhein and Ronald W. Schater , *Digital Signal Processing*, PHI, 1975.

3. Proakis Manodans, *Digital Signal Processing: Principles, Algorithms and Applications*, PHI, 2003.

4. Vijay K. Madisetti, *The Digital Signal Processing Hand Book*, Butterworth-Heinemann, USA, 1999.

5. Vinay K. Ingle, John G. Proaksis, *Digital Signal Processing - A MATLAB Based Approach*, Cengage Learning, 2009.

GE 3.2 Neural Networks

Introduction: Neuron as basic unit of Neurobiology, McCulloch-Pitts model, Hebbian Hypothesis; limitations of single-layered neural networks.

Supervised Learning: Single-layered neural networks, Hopfield-Little model, perceptron rules, Gradient-descent algorithms; Multi-layered neural networks: first order methods, back propagation algorithm, second order methods, RBF networks; Constructive algorithms: single hidden layer algorithms, upstart algorithm, cascade correlation algorithm; Unsupervised Learning: competitive learning, competition through lateral inhibition. Kernel methods and support vector machines: binary classification, multiclass classification, allowing for training errors: soft margin techniques; neural networks and temporal sequences: sequence recognition, sequence generation; applications.

Books Recommended:

1. S. Haykin, Neural Networks: *A Comprehensive Foundation*, 2nd Ed., Prentice Hall,1999.

2. R. Rojas and J. Feldman, *Neural Networks: A Systematic Introduction*, Springer, 1996.

3. C.M. Bishop, *Neural Networks for Pattern Recognition*, Oxford University Press,1995.

4. S.N. Sivanandam, S. Sumathi and S.N. Deepa, *Introduction to Neural Networks using MATLAB 6.0*, Tata McGraw-Hill, 2006.

5. B. Yegnanarayana, *Artificial Neural Networks*, Prentice-Hall of India, 2006.

GE 3.3 Dynamical Systems

Linear Dynamical Continuous Systems: First order equations, existence uniqueness theorem, growth equation, logistic growth, constant harvesting, Planar linear systems, equilibrium points, stability, phase space, n-dimensional linear systems, stable, unstable and center spaces.

Nonlinear autonomous Systems: Motion of pendulum, local and global stability, Liapunov method, periodic solution, Bendixson's criterion, Poincare Bendixson theorem, limit cycle, attractors, index theory, Hartman Grobman theorem, nonhyperbolic critical points, center manifolds, normal forms, Gradient and Hamiltonian systems.

Local Bifurcation: Fixed points, saddle node, pitchfork trans-critical bifurcation, Hopf bifurcation, co-dimension. Discrete systems: Logistic maps, equilibrium points and their local stability, cycles, period doubling, chaos, tent map, horse shoe map. Deterministic chaos: Duffing's oscillator, Lorenz System, Liapunov exponents, routes to chaos, necessary conditions for chaos.

Books Recommended

1. M.W. Hirsch, S. Smale, R.L. Devaney, *Differential Equations, Dynamical Systems and an Introduction to Chaos*, Academic Press, 2008.

2. S.H. Strogatz, *Nonlinear Dynamics and Chaos*, Westview Press, 2008.

3. M. Lakshmanan, S. Rajseeker, *Nonlinear Dynamics*, Springer, 2003.

4. L. Perko, *Differential Equations and Dynamical Systems*, Springer,1996.

5. J.H. Hubbard, B.H. West, *Differential equations: A Dynamical Systems Approach*, Springer-Verlag, 1995.

6. D. Kaplan, L. Gloss, *Understanding Nonlinear Dynamics*, Springer, 1995.

7. S. Wiggins, *Introduction to Applied Nonlinear Dynamical Systems and Chaos*, Springer-Verlag, 1990.

GE 4.1 Industrial Mathematics

Medical Imaging and Inverse Problems: The content is based on Mathematics of X-ray and CT scan based on the knowledge of calculus, elementary differential equations complex numbers and matrices.

Introduction to Inverse problems: Why should we teach Inverse Problems? Illustration of Inverse problems through problems taught in Pre-Calculus, Calculus, Matrices and Differential Equations. Geological anomalies in Earth's interior from measurements at its surface (Inverse problems for Natural disaster) and Tomography.

X-ray: Introduction, X-ray behavior and Beers Law (The fundament question of image construction) Lines in the place.

Radon Transform: Definition and Examples, Linearity, Phantom (Shepp- Logan Phantom - Mathematical phantoms).

Back Projection: Definition, properties and examples.

CT Scan: Revision of properties of Fourier and inverse Fourier transforms and applications of their properties in image reconstruction. Algorithms of CT scan machine. Algebraic reconstruction techniques abbreviated as ART with application to CT scan.

Books Recommended

1. Timothy G. Feeman, *The Mathematics of Medical Imaging, A beginners guide*, Springer Under graduate Text in Mathematics and Technology, Springer 2010.

2. C.W. Groetsch, *Inverse Problems*, Activities for undergraduates, the Mathematical Association of America, 1999.

3. Andreas Kirsch, *An Introduction to the Mathematical Theory of Inverse Problems*, 2nd Ed., Springer, 2011.

GE 4.2 Statistical Techniques

Probability: Basic concepts and definitions (Classical and Axiomatic definition), random variable, probability density function, probability mass function, distribution function and their properties, mathematical expectation, conditional expectation, moment generating function, Characteristic Function, Chebyshev's inequality.

Various discrete and continuous probability distributions: Uniform (continuous and discrete), Binomial, Negative Binomial, Poisson, Exponential, Erlang, Gamma, Normal, t distribution and F-distribution, Bivariate normal distribution (Marginal and Conditional distributions), Weak Law of Large Numbers, Central Limit Theorem. Simple random sampling with and without replacement, Random number generation using inverse transformation technique (exponential distribution, gamma distribution)

Statistical Testing and Estimation Techniques: Properties of good estimator- unbiasedness, consistency, sufficiency, completeness, efficiency; Minimum variance unbiased estimators, Cramer Rao Inequality, Method of Maximum likelihood, method of Moments, Confidence Intervals for mean, variance and proportions. Large sample tests for mean and proportion, chi square test for goodness of fit, Tests based on t and F-distributions.

Correlation and Regression: Least square method for curve fitting, multiple regression (three variables only), Partial and multiple Correlation (for three variables only).

Books Recommended

1. V.K Rohtagi and A.K. Saleh, *An Introduction to Probability and Statistics*, 2nd Ed., John Wiley & Sons, 2005.

2. A.M. Goon, M.K. Gupta and T.S. Dasgupta, *Fundamentals of Statistics* (Vol. I), 7th Ed., The World Press Pvt. Ltd., 2000.

3. R.V. Hogg and A.T. Craig, Introduction to Mathematical Statistics, Macmillan Publishing Co. Inc., 1978.

4. Neil A. Weiss, *Introductory Statistics*, 7th Ed., Pearson Education, 2007.

5. A.M. Goon, M.K. Gupta and T.S. Dasgupta, *An Outline of Statistical Theory* (Vol. II), 2nd Ed., The World Press Pvt. Ltd., 2000.

GE 4.3 Modeling and Simulation

What is Mathematical Modeling? History of Mathematical Modeling, latest development in Mathematical Modeling, Merits and Demerits of Mathematical Modeling.

Introduction to difference equations, Non-linear Difference equations, Steady state solution and linear stability analysis. Introduction to Discrete Models, Linear Models, Growth models, Decay models, Newton's Law of Cooling, Bank Account Problem and mortgage problem, Drug Delivery Problem, Harrod Model of Economic growth, War Model, Lake pollution model, Alcohol in the bloodstream model, Arm Race models, Linear Prey-Predator models, Density dependent growth models with harvesting, Numerical solution of the models and its graphical representation using EXCEL.

Introduction to Continuous Models, Carbon Dating, Drug Distribution in the Body, Growth and decay of current in a L-R Circuit, Horizontal Oscillations, Vertical Oscillations, Damped Force Oscillation, Dynamics of Rowing, Combat Models, Mathematical Model of Influenza Infection (within host), Epidemic Models (SI, SIR, SIRS, SIC), Spreading of rumour model, Steady State solutions, Linearization and Local Stability Analysis, logistic and gomperzian growth, prey-predator model, Competition models, Numerical solution of the models and its graphical representation using EXCEL.

Fluid flow through a porous medium, heat flow through a small thin rod (one dimensional), Wave equation, Vibrating string, Traffic flow, Theory of Car-following, Crime Model, Linear stability Analysis: one and two species models with diffusion, Conditions for diffusive instability with examples.

Books Recommended

1. B. Albright, *Mathematical Modeling with Excel*, Jones and Bartlett Publishers, 2010.

2. F.R. Marotto, *Introduction to Mathematical Modeling using Discrete Dynamical Systems*, Thomson Brooks/Cole, 2006.

3. J.N. Kapur, *Mathematical Modeling*, New Age International, 2005.

4. B. Barnes and G. R. Fulford, *Mathematical Modelling with Case Studies*, CRC Press, Taylor and Francis Group, 2009.

5. L. Edsberg, *Introduction to Computation and Modeling for Differential Equations*, John Wiley and Sons.